Z. Angew. Math. Phys. (2020) 71:77

https://doi.org/10.1007/s00033-020-01298-5

Zeitschrift für angewandte Mathematik und Physik ZAMP

Solving procedure for the dynamics of charged particle in variable (time-dependent) electromagnetic field

Sergey Ershkov and Dmytro Leshchenko

Abstract. In this challenging analytic survey, we present a new approach for solving equations of the dynamics of a charged particle (describing its motion in variable, time-dependent electromagnetic field) which has been applied earlier in various fields of mechanics for solving equations of hydrodynamics, Euler–Poisson equations of rigid body rotation and even in celestial mechanics (solution of equations of small body's motion in the CR3BP problem near librations points): a new type of the solving procedure is implemented in all of these equations as well as in case of solving momentum equation for the aforementioned dynamics of a charged particle, determined by Lorentz force in non-relativistic case. Meanwhile, in each case the system of momentum equations has been successfully solved analytically. The main result of the current research should be formulated as follows: the analytic algorithm is pointed out for solving momentum equation, which has been reduced to the analytical solution of three nonlinear ODEs with respect to the components of velocity of the particle. Moreover, absolutely new *partial* analytical solutions have been obtained for the special cases of magnetic field (with zero electric field's components). In addition to this, we conclude that system of Lorentz momentum equations for dynamics of a charged particle has not the analytical presentation of solution in case of nonzero, time-dependent electric field.

Mathematics Subject Classification. 70E40 (Integrable cases of motion).

Keywords. Lorentz force, Charged particle, Poisson equations, *Riccati* equation.

1. Introduction, equations of motion

Lorentz force [1] plays a significant crucial role in describing the dynamics of velocity of charged particles, moving under the action of electromagnetic field (including dynamics of solar wind near the magnetic shield of the planets); this is extremely nonlinear problem even in non-relativistic case insofar. It is worth to note that only a few cases of analytical or semi-analytical solution are known in the history of electromagnetic theory, including trivial case of zero electric field along with constant magnetic field (which means a circular motion of a particle) as well as elegant Alfven's solution for solar wind in MHD theory [2].

The problem of motion of a charged particle under a Lorentz force has been studied in the past under various versions (e.g., Störmer's problem, the magnetic-binary problem, as well as in various models of Celestial Mechanics, Geophysical Sciences and Plasma Physics), and there is a rich and extended international bibliography. In the *future* research, we will restrict ourselves in presenting a new analytical technique for solving equations of motion of a charged particle under the action of the Lorentz force with additional taking into consideration the *Joule effect* which corresponds to the case of motion of a charged particle with additional influence of energy dissipation phenomenon or the heating effect during the particle's motion along it's trajectory. We should especially note that in the current research the ideal problem is considered, without losses of energy for the particle due to appropriate dissipative processes via its interacting with surrounding plasma medium governed by outer electromagnetic field in the current continuum. As for the complete introduction to the problem under the current consideration,

Published online: 10 April 2020