

## A New Solving Procedure for the Kelvin–Kirchhoff Equations in Case of Falling a Rotating Torus

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We present a new solving procedure in this paper for Kelvin–Kirchhoff equations, considering the dynamics of falling with *rigid* rotating torus in an ideal incompressible fluid, assuming additionally that the dynamical symmetry of rotation for the rotating body,  $I_1 = I_2$ .

The fundamental law of angular momentum conservation is used for the aforementioned solving procedure. The system of *Euler* equations for the dynamics of torus rotation is explored for an analytic way of presentation of the approximated solution (where we consider the case of laminar flow at slow regime of torus rotation). The second finding is the Stokes boundary layer phenomenon on the boundaries of the torus also assumed during the formulation of basic Kelvin–Kirchhoff equations (for which the analytical expressions for the components of fluid’s torque vector  $\{T_2, T_3\}$  were obtained earlier). The results for calculating the components of angular velocity  $\{\Omega_i\}$  should then be used for full solving the momentum equation of Kelvin–Kirchhoff system. The trajectories of motion can be divided into, preferably, three classes: zigzagging, helical spiral motion, and *the chaotic regime* of oscillations.

*Keywords:* Kelvin–Kirchhoff equations; Euler equations; chaotic regime.

### 1. Introduction, Equations of Motion

Kelvin–Kirchhoff equations describe the dynamics of rigid body motion in an ideal fluid [Kirchhoff, 1877; Ershkov *et al.*, 2020a], expressing the conservation of linear and angular momentum for the coupled fluid-to-body interaction problem (and *vice versa*).

The variability of the aforementioned motions observed in the trajectories during falling or ascending of particles (or of bodies) can be divided into, preferably, three classes of motions: zigzagging, helical spiral motion, and the chaotic regime of oscillations. We should mention the recent comprehensive works [Mathai *et al.*, 2018; Pan *et al.*, 2019; Ern *et al.*, 2012; Novikov & Shmel’tser, 2011].

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