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Solving procedure for the Kelvin–Kirchhoff equations in case of buoyant (or the falling) ellipsoid of rotation

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ABSTRACT

We have presented in this communication a new solving procedure for Kelvin–Kirchhoff equations, considering the dynamics of rising the *quasi-rigid* ellipsoid of rotation in an ideal incompressible fluid (up to the surface), assuming additionally the dynamical symmetry of rotation for the rising body, $I_1 = I_2$.

Fundamental law of angular momentum conservation has been used for the aforementioned solving procedure. The system of *Euler* equations for dynamics of *non-rigid* ellipsoid rotation has been explored in regard to the existence of an analytic way of presentation for the approximated solution (where we suppose that components of fluid's torque vector $\{T_i\}$ are approximately proportional to the appropriate components of angular velocity $\{\Omega_i\}$). The results of calculations for the components of angular velocity $\{\Omega_i\}$ should then be used for solving momentum equation of Kelvin–Kirchhoff system. Thus, the full system of equations of Kelvin–Kirchhoff problem has been explored with respect to the existence of an analytic way of presentation of the general solution.

The last but not least, we have pointed out the 1-st integral of Kelvin–Kirchhoff system under the aforesaid additional assumption for fluid's torque vector $\{T_i\}$ when $\{I_i\}$ = *const* (but without the additional restriction of dynamical symmetry onto the form of the rising body, $I_1 = I_2$).

1. Introduction, equations of motion

Kelvin–Kirchhoff equations describe the dynamics of rigid body motion in an ideal fluid [1,2], expressing the conservation of linear and angular momentum for the coupled fluid-to-body interaction problem (and *vice versa*). Even in such simplified ideal formulation (excepting motions of e.g. the elastic *non-rigid* bodies in a viscous fluid) this classical problem of mechanics has not, nevertheless, been clearly solved up to the last decade; meanwhile, various approaches to the aforesaid problem have been attracting a lot of best minds in mechanics during last 300 years.

One of the famous scientists in the field of physics, mechanics and astronomy, Sir Isaac Newton observed variability of these phenomena are intriguing; the variability observed in the trajectories during falling or ascending of particles (or of bodies) can be divided into, preferably, 3 classes of motions (along with switching between them): zigzagging, helical spiral motion, and the chaotic trembling or regime of oscillations. Also, there are a larger amount of recent works, concerning significant achievements with respect to solution of this problem, such comprehensive researches should be mentioned accordingly [3–6] (in [6] the special case of particles rising in a viscous fluid was considered).

Most of researchers have come to a reasonable conclusion regarding a key importance of the two governing parameters for adequate describing the dynamics of *quasi-rigid* ellipsoidal or spherical body's motion in an ideal fluid: the *quasi-rigid* particle's density ρ_p relative to the density of fluid ρ_f ($\Gamma \equiv \rho_p/\rho_f$, $\Gamma < 1$), and its Galileo number $\mathrm{Ga} \equiv \frac{1}{\nu}\sqrt{g\,D^3\,(1-\Gamma)}$($g$ is the acceleration due to gravity, D is the ellipsoidal particle rotational diameter, and ν is the kinematic viscosity of the fluid in the viscous Stokes boundary layer on the boundaries of the particle). In [3] the additional governing parameter has been taken into consideration by researchers, namely, the modulating a *quasi-spherical* particle's moment of inertia (MoI). Indeed, non-uniform distribution of inertia properties inside the rigid sphere due to variations of density of the *quasi-rigid* particle (inside its boundaries) would be causing

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