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ORIGINAL ARTICLE

Solving procedure for 3D motions near libration points in CR3BP

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Abstract In a novel approach for solving the equations of the Circular Restricted Three-Body Problem (CR3BP) first formulated in Ershkov (Acta Mech. 228(7):2719–2723, 2017a), we apply in this communication a procedure for solving the Euler-Poisson equations for the momentum equations of the CR3BP near the libration points for uniformly rotating planets having inclined orbits in the solar system with respect to the orbit of the Earth. The system of equations of the CR3BP has been explored with regard to the existence of an analytic way of presentation of the approximated solution in the vicinity of libration points. A new and elegant ansatz is suggested in this publication, whereby, in solving, the momentum equation is reduced to a system of three linear ordinary differential equations of first order in regard to the three components of the velocity of the infinitesimal mass m (dependent on time t). Under this premise, a proper elegant partial solution has been obtained due to the invariant dependence between temporary components of the solution. We conclude that the system of CR3BP equations does not have the analytical presentation of the solution (in quadratures) even in the vicinity of the libration points except of the generalized Jacobi integral.

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1 Introduction, equations of motion

The equations of motion of the restricted three-body problem (R3BP) describe the dynamics of infinitesimal mass m under the action of gravitational forces affected by two celestial bodies of giant masses (in this problem M_{Sun} and m_{planet}, $m_{planet} < M_{Sun}$), which are rotating around their common center of masses on Kepler trajectories. The aforesaid infinitesimal mass m is supposed to be moving (as a first approximation) inside the *restricted* region of space around the planet of mass m_{planet} or inside the so-called *Hill sphere* (Ershkov and Shamin 2018a) (where a_p is the semimajor axis of the planet):

$$r_H = a_p \cdot \left(\frac{m_{planet}}{M_{Sun}}\right)^{\frac{1}{3}} \qquad (*)$$

It is worth to note that a large amount of previous and recent results concerning analytical development with respect to these equations exist; the fundamental work in Arnold (1978), Lagrange (1873), Szebehely (1967), Duboshin (1968), Bruns (1887), Chernikov (1970) should be mentioned particularly.

We should especially emphasize the theory of orbits, which was developed in profound work (Szebehely 1967) for the case of the Circular Restricted Problem of Three Bodies (CR3BP) (primaries M_{Sun} and m_{planet} are rotating around their common center of masses on *circular* orbits). According to Szebehely (1967), the equations of motion of the infinitesimal mass m should be presented in the co-