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PERTURBED MOTIONS OF A SPHEROID WITH CAVITY CONTAINING A VISCOUS FLUID

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ABSTRACT

A satellite or a spacecraft in its motion about the centre of mass is affected by the moments of forces of various physical nature. These motions may have various causes, for example, the presence of fluid in the cavities in the body (for example, liquid fuel or oxidizer in the tanks of a rocket). Therefore, there is a need to study the problems of the dynamics of bodies with cavities containing a viscous fluid, to calculate the motion of spacecrafts about the centre of mass, as well as their orientation and stabilization.

We consider a free motion in space of a rigid body with a cavity filled with highly viscous fluid relative to the centre of inertia. The constant tensor $\boldsymbol{P}(P_{ij})$ depends only on the shape of the cavity [1]. The book can be used by researchers working in the field of mechanics and applied mathematics, engineers working in attitude dynamics of spacecraft. In the case of a spherical cavity of radius b: $P_{ij} = P_0\ \delta_{ij}$, $P_0 = 8\pi b^7/525$, $i,j=1,2,3$. Here, δ_{ij} is a Kroneker symbol. Denote by A, B, C the principal moments of inertia of the system and by p, q, r the projections of the absolute angular velocity $\boldsymbol{\omega}$ on the principal central axes of inertia. Equations of motion are written in the projections on the principal central axes of inertia (the dot denotes the derivative with respect to time):

$$A\dot{p} + (C-B)qr = \frac{\rho P_0}{\nu ABC}\, p\left[C(A-C)(A+C-B)r^2 + B(A-B)(A+B-C)q^2\right] \qquad (1)$$

The remaining equations are obtained from (1) by cyclic permutation of symbols A, B, C and p, q, r. Suppose that the principal central moments of inertia of a rigid body are close to one another and they are represented in the form:

$$A = J_0 + \varepsilon A',\ B = J_0 + \varepsilon B',\ C = J_0, \qquad (2)$$

where $0 < \varepsilon << 1$ is a small parameter.

We obtain the perturbed Euler system after transformation of the system (1) and transfer for slow time $\tau = \varepsilon t$ (later the dot denotes the derivative with respect to τ):

$$\frac{dp}{d\tau}=\frac{B'}{J_0}\left(1-\varepsilon\frac{A'}{J_0}\right)qr+\varepsilon f_p(p,q,r),\ \ p(0)=p_0$$
$$\frac{dq}{d\tau}=\frac{A'}{J_0}\left(-1+\varepsilon\frac{B'}{J_0}\right)rp+\varepsilon f_q(p,q,r),\ \ q(0)=q_0 \tag{3}$$
$$\frac{dr}{d\tau}=\frac{\varepsilon}{J_0}(A'-B')qp+\varepsilon f_r(p,q,r),\quad r(0)=r_0$$

Here r is a slow variable. The system (3) is an essentially nonlinear system. The perturbations are introduce in (3):

$$\varepsilon f_p(p,q,r)=\frac{\rho P_0 p}{\nu J_0^3}\left\{A'[J_0-\varepsilon(A'+2B')]r^2+(A'-B')[J_0-\varepsilon(A'-B')]q^2\right\}$$
$$\varepsilon f_q(p,q,r)=\frac{\rho P_0 q}{\nu J_0^3}\left\{(B'-A')[J_0-\varepsilon(B'-A')]p^2+B'[J_0-\varepsilon(2A'+B')]r^2\right\}$$
$$\varepsilon f_r(p,q,r)=\frac{\rho P_0 r}{\nu J_0^3}\left\{B'[\varepsilon(2A'-B')-J_0]q^2+A'[\varepsilon(2B'-A')-J_0]p^2\right\}$$

The solution of nonperturbed system (3) when $\varepsilon = 0$, $\nu^{-1} = 0$ has a form:

$$p=a\sin\varphi,\quad q=a\sqrt{\frac{A'}{B'}}\cos\varphi,\ a=\sqrt{p_0^2+\left(\dot{p}_0/w\right)^2}$$

Here a is amplitude and φ is phase, $w=r_0/J_0\sqrt{A'B'}$, $A'B'>0$.

We find the system for slowly varying variables after some transformations and averaging with respect to the phase φ

$$\dot{x}=2\eta\, x(\alpha x+\beta y),\quad \dot{y}=\eta\gamma xy \tag{4}$$

Here $x=a^2$, $y=r^2$; α, β, γ, η depends on ρ, P_0, ν, J_0, A', B', ε.

Divide the first equation (4) by second one after some conversions we can be reduce to the equation

$$\frac{dz}{d\theta}=(\tilde{\alpha}-1)z+\tilde{\beta} \tag{5}$$

Here $z=x/y$, $\tilde{\alpha}=2\alpha/\gamma$, $\tilde{\beta}=2\beta/\gamma$, $\theta=\ln y$.

We find the first integral of this equation

$$x=\frac{\tilde{\beta}}{1-\tilde{\alpha}}y+C_1y^{\tilde{\alpha}},\qquad C_1=\text{const}. \tag{6}$$

System (4) was solved numerically for the initial conditions x(0) = 1, y(0) = 1. The plots of the changing magnitudes a^2 and r^2 of the equatorial and axial components of the angular velocity vector of rigid body are constructed.

It is noted that graphics of function $x=a^2$ and $y=r^2$ are decreasing (Fig. 1, Fig. 2), asymptotically approaching zero and a stationary value 0.97 respectively.

We obtained the system of equations of motion in the standard form which refined in square-approximation by small parameter. The Cauchy problem for a system determined after averaging was analyzed. The evolution of the motion of a rigid body is described by the solutions which obtained as a result of asymptotic, analytical and numerical calculations over an infinite time interval.

In our paper we are investigating the model which represents a certain natural-scientific interest for the dynamics of figure of the Earth.

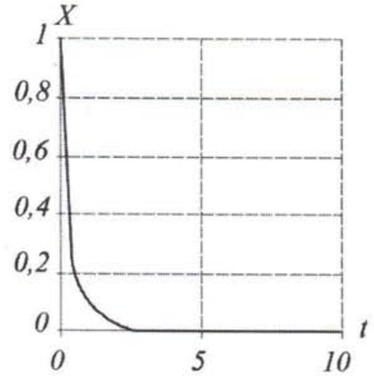

Figure 2

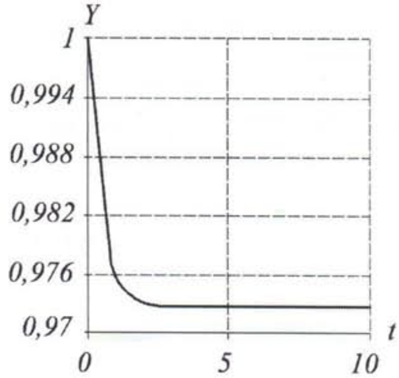

Figure 2

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