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THE GENERALISED DYNAMICAL PROBLEM OF THERMOELASTICITY FOR THE HOLLOW SPHERE

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Abstract. The generalized dynamical problem of thermoelasticity for the hollow sphere has been considered. It is supposed that the propagation of the temperature is symmetric with respect to the center of the sphere. The temperature and stresses are equal to zero at initial time. Radial displacements and the heat flow are given at surfaces of the sphere. Due to complexity of the statement of such boundary-value problems the most of the published solutions have been obtained as a result of using approximate methods to expand these solutions in the form of series for small and large times. In this work the closed exact solution of the given problem has been received. In introducing thermoelastic potentials which are connected with the temperature and the displacement (quasistatic and dynamical potentials) the problem is reduced to the solution of the system of two differential equations of the second and the third order in partial derivatives with respect to the time and the space coordinate. To improve the convergence of series at the boundary of the sphere potentials are represented as the sum of two functions. One of these functions is the solution of the system of equations with homogeneous conditions and the other is with nonhomogeneous conditions. Applying the finite integral transform on the space coordinate we come to the system of ordinary differential equations with respect to time potentials. The eigenfunctions of the corresponding Sturm – Liouvill problem are the kernels of the finite integral transforms. The asymptotic formula for eigenvalues depending on the parameter which tends to infinity is obtained. Laplace transform is used to get the solution of derived equations. It is analyzed the special case when the harmonic displacement at the surface is given and the heat flow equals zero. It was shown if the coupling constant tends to zero, then the temperature diminishes to zero. In this case the temperature disturbances can be large is the coupling constant is however small and the wave number is close to the eigenvalue of the corresponding Sturm – Liouvill problem (resonance conditions).

Keywords: coupling constant, dynamical, displacement, eigenvalue, potential, stress, thermoelasticity, transform.

Introduction. In the modern technology one has encountered extremely high heating rates and it is necessary to examine the role of inertia. In the uncoupled problem velocities are readily associated with a strain wave front travelling at the material acoustic velocity. In the coupled theory, no such simple distinction of the wave speeds is possible. Thus, the solution of such problems is important and represents considerable problems. This is the reason that only approximate methods have been used to solve such problems. In this paper the closed exact solution of the generalized dynamical problem of thermoelasticity for the hollow sphere has been constructed.

The generalized dynamical theory has been formulated by Lord H.W. and Shulman Y. [1] by including the time needed for acceleration of the heat flow in the heat conduction equation. The generalized dynamical theory of thermoelasticity takes into account the coupling effect between the temperature and the strain rate. The dynamic problem of thermoelasticity in the uncoupled theory in a hollow elastic sphere was considered by Tsui T., and Kraus H. [2]. After passing to appropriate limits, the results were compared to the results of the previous analyses of slender and massive spherical regions. Norwood F.R. and Warren W.E. [3] analyzed the wave propagation in the generalized dynamical theory of thermoelasticity. Wave front and longtime approximations have been obtained. A dynamical problem for a spherical cavity in an infinite medium in the generalized theory of thermoelasticity has been considered by Wadhawan M.C. [4]. An approximate solution has been obtained by perturbation expansion. In paper [5] the generalized dynamical problem of thermoelasticity for an isotropic infinite cylinder has been solved by approximate techniques. Hetnarski R.B. and Ignaczar I. [6] have considered the response of semi-space to a short laser pulse in the case of the generalized thermoelasticity. The exact closed solution of the generalized dynamical problem of thermoelasticity for the hollow cylinder has been received in paper [7].

Purpose and tasks. Our aim is to find the closed exact solution of the generalized dynamical problem of thermoelasticity for the hollow sphere when the radial displacements and the heat flow are given at the surfaces of the sphere. Using the finite integral transforms on the coordinate and Laplace transform on time the solution of this problem has been represented in the form of series.

Methods of research. Due to complexity of the initial value problems of a hyperbolic termoelasticity most of the solutions to these problems so far have been obtained in one-dimensional case and by using approximate methods, such as small-time and large-time expansion methods, ray series expansions method, or finite element procedure. All these approximate solutions suffer from a number of limitation imposed on the range of independent variables and parameters involved at which they are acceptable both mathematically and physically. Introducing the thermoelastic potentials by formulas: $\Phi(r,t) = r\mathrm{T}(r,t), u(r,t) = \frac{\partial}{\partial r}\left(\frac{\Psi(r,t)}{r}\right)$, the given problem is reduced to the system of partial differential equations. To get the closed exact solutions of these equations the following method which consists in applying finite transforms on the space coordinate is proposed. The kernels of this finite integral transform are the functions: $Wn(r) = A_n \cos\gamma_n r + Bn\sin\gamma_n r.$ The eigenvalues γ_n satisfy the equation: $tg\gamma_n(R_2 - R_1) = \frac{(R_2 - R_1)\gamma_n}{1 + R_2 - R_1\gamma_n^2}$. Since the eigenvalues γ_n tend to infinity as $n \to \infty$ it is possible to use asymptotic methods to get such values of γ_n, $\gamma_n^2 = \frac{\lambda}{R_2 - R_1} n^2 + \mathrm{O}(n)$. The boundary conditions to find A_n and B_n are given accordingly the boundary conditions for temperature $\mathrm{T}(r,t)$ and the displacement $u(r,t)$, that is $\frac{\partial}{\partial r}\left(\frac{w_n}{r}\right) = 0$ if $r = R_j(i = 1, 2)$. On applying the Laplace transform to the system of the equations with respect to time and solving this system and performing the inverse Laplace transforms we get the solution of this system in the form of series on the eigenfunctions of the corresponding Sturm- Liouvill problem.

Basic Results. Let us consider the hollow isotropic sphere with interior radius R_1 and exterior radius R_2. We suppose that the propagation of the temperature is symmetric with respect to the center of the sphere. Then we have the equation for the definition of the temperature field [8]:

$$\Delta T - \frac{1}{a_T}\frac{\partial T}{\partial t} - \eta\dot{\varepsilon}_{kk} = 0, \tag{1}$$

where $\Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}$, $\varepsilon_{kk} = \frac{\partial u}{\partial r} + \frac{2}{r}u$; ε_{kk} is a solid volume deformation, α_T is the coefficient of linear thermal expansion, $\eta = \frac{2G(1+\nu)}{(1-2\nu)\lambda_T}\alpha_T T_0$ is the coupling constant, ν is Poisson's ratio, a_T is the thermal diffusivity, λ_T is the coefficient of thermal conductivity, G is the modulus of elasticity.

The equation of motion can be written in the form [8]:

$$\Delta u - \frac{2u}{r^2} - \frac{1}{c_1^2}\frac{\partial^2 u}{\partial t^2} = m\frac{\partial T}{\partial r}, \tag{2}$$

where $u(r,t)$ is the displacement, $m = \frac{1+\nu}{1-\nu}\alpha_T, c_1 = \sqrt{\frac{2G}{(1-2\nu)\rho}}$ is the velocity of longitudinal vaves, ρ is the density.

Equations (1) and (2) are subjected to the following initial and boundary conditions:

$$T(r,t) = u(r,t) = \frac{\partial u}{\partial t} = 0 \text{ if } t = 0,\ \frac{\partial T}{\partial r} = g_j(t),\ u(r,t) = h_j(t) \text{ if } r = R_j (j = 1,2), \tag{3}$$

Introducing the thermoelastic potentials by formulas:

$$\Phi(r,t) = rT(r,t), u(r,t) = \frac{\partial}{\partial r}\left(\frac{\Psi(r,t)}{r}\right) \tag{4}$$

and substituting them into equations (1) – (3) we get:

$$\frac{\partial^2 \Phi}{\partial r^2} - \frac{1}{a_T}\frac{\partial \Phi}{\partial t} - \eta\frac{\partial^3 \Psi}{\partial t \partial r^2} = 0,\ \frac{\partial^2 \Psi}{\partial r^2} - \frac{1}{c_1^2}\frac{\partial^2 \Psi}{\partial t^2} = m\Phi, \tag{5}$$

$$\Phi(r,t) = \Psi(r,t) = \frac{\partial \Psi}{\partial t} = 0 \text{ if } t = 0;\ \frac{\partial}{\partial r}\left(\frac{\Phi}{r}\right) = g_j(t),\ \frac{\partial}{\partial r}\left(\frac{\Psi}{r}\right) = h_j(t) \text{ if } r = R_j\ (j = 1,2) \tag{6}$$

In this case we can get the closed exact solution of problem (5) – (6) in the form of series on some eigenfunctions. The solution of problem (5), (6) can be thought as follows:

$$\Phi(r,t) = \Phi^*(r,t) + \varphi(r,t), \Psi(r,t) = \Psi^*(r,t) + \psi(r,t) \tag{7}$$

where: $\varphi(r,t) = A(t)r^2 + B(t), \psi(r,t) = C(t)r^2 + D(t), A(t) = \frac{1}{\Delta}\left(\frac{g_2(t)}{R_1^2} - \frac{g_1(t)}{R_2^2}\right),$

$$B(t) = \frac{1}{\Delta}(g_2(t) - g_1(t)),\ C(t) = \frac{1}{\Delta}\left(\frac{h_2(t)}{R_1^2} - \frac{h_1(t)}{R_2^2}\right),\ D(t) = \frac{1}{\Delta}(h_2(t) - h_1(t)), \tag{8}$$

$$\Delta = \frac{1}{R_1^2 R_2^2}\left(R_2^2 - R_1^2\right).$$

The substitution of (7) into (5), (6) yields:

$$\frac{\partial^2 \Phi^*}{\partial r^2} - \frac{1}{a_T}\frac{\partial^2 \Phi^*}{\partial t^2} - \eta\frac{\partial^3 \Psi^*}{\partial t \partial r^2} = f_1(r,t), \frac{\partial^2 \Psi^*}{\partial r^2} - \frac{1}{c_1^2}\frac{\partial^2 \Psi^*}{\partial t^2} = m\Phi^* + f_2(r,t) \tag{9}$$

$$\Phi^*(r,o) = -\varphi(r,o), \Psi^*(r,o) = -\Psi(r,o),\qquad \frac{\partial \Psi^*(r,o)}{\partial t} = -\frac{\partial \psi(r,0)}{\partial t}; \tag{10}$$

$$\frac{\partial}{\partial r}\left(\frac{\Phi^*}{r}\right) = \frac{\partial}{\partial r}\left(\frac{\Psi^*}{r}\right) = 0 \text{ if } r = R_j, \text{ where} \tag{11}$$

$$f_1(r,t) = 2\eta C'(t) + \frac{1}{a_T}\frac{\partial \varphi}{\partial t} - 2A(t),\qquad f_2(r,t) = m\varphi(r,t) + \frac{1}{c_2^2}\frac{\partial^2 \psi}{\partial t^2} - 2C(t).$$

We solve problem (9) – (11) by using the finite integral transform [9]. To do this we find the solution of the following Sturm – Lioville problem:

$$\frac{d^2 W}{dr^2} + \gamma_n^2 W = 0, \tag{12}$$

$$\frac{\partial}{\partial r}\left(\frac{W}{r}\right) = 0 \text{ if } r = R_j (j = 1,2).$$

We obtain $W_n(r) = A_n \cos\gamma_n r + B_n \sin\gamma_n r$,

where: $A_n = \dfrac{R_1\gamma_n - tg\gamma_n R_1}{1 + R_1\gamma_n tg\gamma_n R_1} B_n$.

The coefficient B_n is defined from the condition:

$$\int_{R_1}^{R_2} W_n^2(r)dr = 1.$$

The eigenvalues γ_n satisfy the equation:

$$tg\gamma_n(R_2 - R_1) = \frac{(R_2 - R_1)\gamma_n}{1 + R_1 R_2 \gamma_n^2}. \tag{13}$$

Since the eigenvalues γ_n tend to infinity as $n \to \infty$ it is possible to use asymptotic methods. In this case we have:

$$\gamma_n^2 = \frac{\pi}{R_2 - R_1} n^2 + 0(n). \tag{14}$$

The finite transforms and the inversion formulas are:

$$\Phi_n^*(t) = \int_{R_1}^{R_2} \Phi^*(r,t) W_n(r)dr, \Psi_n^*(t) = \int_{R_1}^{R_2} \Psi^*(r,t) W_n(r)dr, \tag{15}$$

$$\Phi^*(r,t) = \sum_{n=1}^{\infty} \Phi_n^*(t) W_n(r)dr,\ \Psi^*(r,t) = \sum_{n=1}^{\infty} \Psi_n^*(t) W_n(r). \tag{16}$$

Applying the transforms (15) to the boundary conditions (11) we arrive at the following system of ordinary differential equations with respect to $\Phi_n^*(t)$ and $\Psi_n^*(t)$.

$$\frac{1}{a_T}\frac{d\Phi_n^*}{dt} - \eta\gamma_n^2 \frac{d\Psi_n^*}{dt} + \gamma_n^2 \Phi_n^* = f_{1n}(t), \tag{17}$$
$$\frac{1}{c_1^2}\frac{d^2\Psi_n^*}{dt^2} + \gamma_n^2 \Psi_n^* + m\Phi_n^* = f_{2n}(t).$$

With the initial conditions:

$$\Phi_n^*(0) = -\varphi_n(0),\ \Psi_n^*(0) = -\psi_n(0), \frac{d\Psi_n^*(0)}{dt} = -\frac{d\psi_n(0)}{dt} \tag{18}$$

On employing the Laplace transform [9] to the system of the equations (17) and the initial conditions (18) and solving them we get:

$$\Phi_n^*(t) = \sum_{k=0}^{2} A_{kn} F_{kn}(t) + \int_0^t f_{1n}(\tau)\left(a_T F_{2n}(t-\tau) + a_T c_1^2 \gamma_n^2 F_{on}(t-\tau)\right)d\tau +$$
$$+ \eta a_T c_1^2 \gamma_n^2 \int_0^t f_{2n}(\tau) F_{1n}(t-\tau)d\tau, \Psi_n^*(t) = \sum_{k=0}^{2} B_{kn} F_{kn}(t) + c_1^2 \int_0^t f_{2n}(\tau) F_{1n}(t-\tau)d\tau + \tag{19}$$
$$+ a_T c_1^2 \int_0^t \left(\gamma_n^2 f_{2n}(\tau) - m f_{1n}(\tau)\right) F_{on}(t-\tau)d\tau,$$

where: $F_{on}(t) = \dfrac{e^{-a_n t}}{\Delta_{1n}} + \dfrac{e^{-b_n t}}{\Delta_{2n}} \sin(c_n t + \varphi_{1n})$,

$$F_{1n}(t) = -\frac{a_n e^{-a_n t}}{\Delta_{1n}} + \frac{\rho_n e^{-b_n t}}{\Delta_{2n}}(c_n \sin(c_n t + \varphi_{2n}) - b_n \sin(c_n t + \varphi_{1n})),$$

$$F_{2n}(t)=\frac{a_n^2 e^{-a_n t}}{\Delta_{1n}}-\frac{\rho_n e^{-b_n t}}{\Delta_{2n}}((c_n^2-b_n^2)\sin(c_n t+\varphi_{1n})+2b_n c_n\sin(c_n t+\varphi_{2n})),$$

$$\Delta_{1n}=c_n^2(b_n-a_n)^2,\Delta_{2n}=c_n^4(a_n c_n-b_n c_n)^2,A_{on}=a_T c_1^2\left(\eta\gamma_n^4\psi_n(0)-\frac{\gamma_n^2}{a_T}\varphi_n(0)\right),$$

$$A_{1n}=-\eta a_T\gamma_n^2\psi_n(0),A_{2n}=a_T\eta\gamma_n^2\psi_n(0)-\varphi(0)-a_T\eta\gamma_n^2\frac{d\psi_n(0)}{dt},$$

$$B_{on}=-mc_1^2\varphi_n(0)-a_T\gamma_n^2\psi_n(0)(1+m\eta c_1^2),\quad B_{1n}=-a_T\gamma_n^2\frac{d\psi_n(0)}{dt}+\psi_n(0),$$

$$B_{2n}=-\frac{d\psi_n(0)}{dt},\rho_n=\sqrt{\Delta_{2n}},tg\varphi_{1n}=\frac{c_n}{b_n-a_n},tg\varphi_{2n}=\frac{a_n-b_n}{c_n},$$

$p_1=-a_n,p_{2,3}=-b_n\pm ic_n(a_n>0,b_n>0)$ are the roots of the equation:

$$p^3+a_T\gamma_n^2p^2+c_1^2\gamma_n^2(1-a_Tm\eta)p+a_Tc_1^2\gamma_n^4=0. \tag{20}$$

Finally, the solution of the problem (1) – (3) can be written in the form:

$$T(r,t)=\frac{1}{r}\left(\varphi(r,t)+\sum_{n=1}^{\infty}\Phi_n^*(t)w_n(r)\right),u(r,t)=\frac{\partial}{\partial r}\left(\frac{\psi(r,t)}{r}\right)+\sum_{n=1}^{\infty}\Psi_n^*(t)(rw_n'(r)-w_n(r)). \tag{21}$$

Next, we consider the special case, when the harmonic displacement is given at the surface of the sphere and the heat flow equals zero, that is:

$$\frac{\partial T}{\partial r}=0,\qquad u=u_je^{-iwt}\ if\ r=R_j(1=1,2).$$

In this case we have:

$$T_n^*(t)=\frac{\eta}{\chi_n}\left(l_1\left(\gamma_n^2-\frac{w^2}{c_1^2}\right)-m\gamma_n^2l_{2n}\right)e^{-iwt}, \tag{22}$$

$$\Psi_n^*(t)=\frac{1}{\chi_n}\left(l_{2n}\left(\gamma_n^2-\frac{iw}{a_T}\right)-ml_1\right)e^{-iwt}, \tag{23}$$

where:

$$l_1=\frac{4}{\Delta}\left(\frac{u_2}{R_1}-\frac{u_1}{R_2}\right),\qquad l_{2n}=\frac{1}{c_1^2\Delta}\left(2(R_2u_1-R_1u_2)\int_{R_1}^{R_2}r^2w_n(r)dr+\left(\frac{u_2}{R_1}-\frac{u_1}{R_2}\right)\int_{R_1}^{R_2}w_n(r)dr\right),$$

$$\chi_n=\left(\gamma_n^2-\frac{w^2}{c_1^2}\right)\left(\gamma_n^2-\frac{iw}{a_T}\right)-m\eta\gamma_n^2.$$

It is clear from (22) that $T_n^*(t)=0$ if coupling constant $\eta=0$, and $T_n(t)=0$,but the temperature disturbances can be large if the value $\eta\neq 0$ is however small and the wave number $\frac{w}{c_1}$ is close to γ_n. Indeed, let $\frac{w}{c_1}=\gamma_n$ (resonance condition), then we get from the formula (22):

$$T_n^*(t)=l_{2n}e^{-iwt}. \tag{24}$$

And the amplitude of these vibrations (not depending on η) can reach however large values.

Conclusions. Thus the closed exact solution of the generalized dynamical problem of termoelasticity for the hollow sphere has been constructed. The convergence of the series at the boundary of the sphere has been improved. The special case when the harmonic displacements are given at the surface of the sphere has been considered. It has been shown that the temperature disturbance can be large if the coupling value $\eta \neq 0$ is however small and the wave number $\frac{w}{c_1}$ is closed to the eigenvalue γ_n.

References

1. Lord H.W., Slulman Y. The generalized dynamical theory of termoelasticity. J.Mech. Phy.Solids, vol. 15, no 5, pp. 229, 1967.

2. Tsui T., Kraus H. Thermal stress-wave propagation in hollow elastic. Spheres, J. of the Acounstical Society, vol. 37, no 4, pp. 730-737, 1964.

3. Norwood F.R., Warren E.R. Wave propagation in the generalized dynamical theory of termoelasticity. Quart, Mech, and Applied Math, vol. 22, pp.283-290, 1969.

4. Wadhawan M.C. Radially symmetrical thermoelastic disturbalances in generalized dynamical termoelasticity. Pure and Apll, Geophysics, vol. 99, pp.61-71, 1972.

5. Wadhawan M.C. Thermoelastic response of a cylinder in the generalized denamical theory of thermoelasticity. Pure and Aplpl.Geoph, vol.102, pp.37-50, 1973.

6. Hetharsri R.B., Iynaczar I. Generalized thermoelasticity: response of semi-space to a short laser pulse. J.Thermal stresses, vol.17, pp. 337-396, 1994.

7. Gavdzinski V., El-Sheikh M., Maltseva E. The generalized dynamical problem of thermoelasticity for the hollow cylinder. Visnyk ODABA, Odessa, vol. 17, pp. 56-60, 2005.

8. Novatski V. Dynamicheskye zadachi termoupruhosti [Dynamical problems of thermoelasticity]. Izd. «Mir», p. 256, 1970.

9. Tranter C.I. Integral transforms in mathematical physics. New York.: John Wiley, p.139, 1966.

ВЗАЄМОПОВ'ЯЗАНА ДИНАМІЧНА ЗАДАЧА ТЕРМОПРУЖНОСТІ ДЛЯ ПОЛОЇ СФЕРИ

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Анотація. Розглядається взаємопов'язана динамічна задача термопружності для полої сфери. Передбачається, що поширення температури відбувається симетрично щодо центру сфери. В початковий момент часу температура і напруга рівні нулю. На поверхнях сфери задані радіальні переміщення та тепловий потік. Зважаючи на складність постановки таких крайових задач, більшість опублікованих рішень отримано в результаті використання наближених методів представлення цих рішень у вигляді рядів для малих і великих значень часу. У даній роботі приводиться точне рішення поставленої задачі. Впроваджуючи термопружні потенціали, пов'язані з температурою та переміщенням (квазістатичний та динамічний потенціали), завдання зводиться до розв'язування системи двох диференціальних рівнянь другого та третього порядків у частинних похідних відносно часу та просторових координат. Для поліпшення збіжності рядів на межі сфери потенціали представлені у вигляді суми двох функцій, одна з яких є розв'язком системи рівнянь з однорідними граничними умовами, а інша з неоднорідними. Приймаючи скінченне інтегральне перетворення за просторовою координатою, приходимо до системи звичайних диференціальних рівнянь відносно потенціалів за часом. Ядрами скінченних інтегральних перетворень є власні функції відповідної задачі Штурма-Ліувілля. Отримано асимптотичну формулу для наближення власних значень, коли параметр, від якого вони залежать, прямує до нескінченності. Для розв'язання отриманих рівнянь використовується перетворення Лапласа. Розглядається частинний випадок, коли на поверхні сфери задано осцилюючe переміщення, а тепловий потік дорівнює нулю. Показано, що якщо коефіцієнт зв'язності прямує до нуля, то температура зменшується до нуля. Однак, коливання температури можуть бути великими, навіть при як завгодно малих значеннях коефіцієнта зв'язності, якщо тільки хвильове число близько до власного значення відповідної задачі Штурма-Ліувілля (умова резонансу).

Ключові слова: динамічний, напруга, переміщення, потенціал, стала зв'язності, перетворення, власне значення, термопружність.

ВЗАИМОСВЯЗАННАЯ ДИНАМИЧЕСКАЯ ЗАДАЧА ТЕРМОУПРОГОСТИ ДЛЯ ПОЛОЙ СФЕРЫ

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Аннотация. Рассматривается взаимосвязанная динамическая задача термоупругости для полой сферы. Предполагается, что распространение температуры происходит симметрично относительно центра сферы. В начальный момент времени температура и напряжение равны нулю. На поверхностях сферы заданы радиальные перемещения и тепловой поток. Ввиду сложности постановки таких краевых задач большинство опубликованных решений получено в результате использования приближенных методов представления этих решений в виде рядов для малых и больших значений времени. В данной работе приводится точное решение данной задачи. Введя термоупругие потенциалы, которые связаны с температурой и перемещением (квазистатический и динамический потенциалы), задача сводится к решению системы двух дифференциальных уравнений второго и третьего порядка в частных производных относительно времени и пространственной координаты. Для улучшения сходимости рядов на границе сферы потенциалы представлены в виде суммы двух функций, одна из которых является решением системы уравнений с однородными граничными условиями, а другая с неоднородными. Применяя конечное интегральное преобразование по пространственной координате, приходим к системе обыкновенных дифференциальных уравнений относительно потенциалов по времени. Ядрами конечных интегральных преобразований являются собственные функции соответствующей задачи Штурма-Лиувилля. Получена асимптотическая формула для нахождения собственных значений, когда параметр, от которого они зависят стремиться к бесконечности. Для решения полученных уравнений применяется преобразование Лапласа. Рассмотрен частный случай, когда на поверхности полой сферы задано осциллирующее перемещение, а тепловой поток равен нулю. Показано, что если коэффициент связанности стремится к нулю, то температура уменьшится до нуля. Однако колебания температуры могут быть большими даже при сколь угодно малых значениях коэффициента связности, если только волновое число близко к собственному значению соответствующей задачи Штурма-Лиувилля (условие резонанса).

Ключевые слова: динамический, напряжение, перемещение, потенциал, постоянная связности, преобразование, собственное значение, термоупругость.

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