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Rapid Rotations of a Satellite with a Cavity Filled with Viscous Fluid under the Action of Moments of Gravity and Light Pressure Forces

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Abstract—Rapid rotational motion of a dynamically asymmetric satellite relative to the center of mass is studied. The satellite has a cavity filled with viscous fluid at low Reynolds numbers, and it moves under the action of moments of gravity and light pressure forces. Orbital motions with an arbitrary eccentricity are supposed to be specified. The system, obtained after averaging over the Euler–Poinsot motion and applying the modified averaging method, is analyzed. The numerical analysis in the general case is performed, and the analytical study in the axial rotation vicinity is carried out. The motion in the specific case of a dynamically symmetric satellite is considered.

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1. PROBLEM FORMULATION

We consider the satellite motion relative to the center of mass under an effect of the moment of light pressure forces in the gravitational field. The body contains a cavity fully filled with highly viscous homogeneous fluid. Rotations are considered within the model of a quasi-solid body, whose center of mass moves over the specified, fixed elliptical orbit around the Sun [1]. The problems of dynamics, generalized and complicated by accounting for various disturbing factors, remain rather topical till now. The studies of rotational motions of bodies relative to the center of mass under an effect of disturbing moments of forces of various nature (gravitational, light pressure, influence of a cavity filled with viscous fluid, etc.), close to that presented below, can be found in papers [1–15].

Let us introduce three Cartesian coordinate systems, whose origin is placed at the satellite's center of inertia [2, 3]. The coordinate system Ox_i $(i = 1,2,3)$ moves forward together with the center of inertia: the Ox_1 axis is parallel to the radius vector of the orbit's perihelion, the axes Ox_2 and Ox_3 are parallel to the vector of velocity of the satellite's center of mass at the perihelion and to the normal to the orbital plane, respectively. The coordinate system Oy_i $(i = 1,2,3)$ is associated with the angular momentum vector $\mathbf{G}$. The Oy_3 axis is directed along the angular momentum vector $\mathbf{G}$, the Oy_2 lies in the orbital plane (i.e., in the plane Ox_1x_2), and the Oy_1 axis lies in the plane Ox_3y_3: it is directed so that vectors $\mathbf{y}_1$, $\mathbf{y}_2$, $\mathbf{y}_3$ form the right-hand triple [2–4]. The axes of the coordinate system Oz_i $(i = 1,2,3)$ are associated with the principal axes of inertia of a solid body. The mutual position of principal axes of inertia and axes Oy_i are determined by the Euler angles. Here, the direction cosines α_{ij} of axes Oz_i relative to the system Oy_i are expressed in terms of the Euler angles φ, ψ, θ by well-known formulas [2]. The position of the angular momentum vector $\mathbf{G}$ relative to its center of mass in the coordinate system Ox_i is determined by angles λ and δ, as is shown in [2–4].

The equations of motion of a body relative to its center of mass are written in the form [3]:

$$\frac{dG}{dt} = L_3, \quad \frac{d\delta}{dt} = \frac{L_1}{G}, \quad \frac{d\lambda}{dt} = \frac{L_2}{G\sin\delta},$$
$$\frac{d\theta}{dt} = G\sin\theta\sin\varphi\cos\varphi\left(\frac{1}{A_1} - \frac{1}{A_2}\right) + \frac{L_2\cos\psi - L_1\sin\psi}{G},$$

$$\frac{d\varphi}{dt} = G\cos\theta\left(\frac{1}{A_3} - \frac{\sin^2\varphi}{A_1} - \frac{\cos^2\varphi}{A_2}\right) \qquad (1.1)$$

$$+\frac{L_1\cos\psi + L_2\sin\psi}{G\sin\theta},$$

$$\frac{d\psi}{dt} = G\left(\frac{\sin^2\varphi}{A_1} + \frac{\cos^2\varphi}{A_2}\right)$$
$$-\frac{L_1\cos\psi + L_2\sin\psi}{G}\text{ctg}\theta - \frac{L_2}{G}\text{ctg}\delta.$$

Here, L_i are the moments of applied forces relative to the axes Oy_i, G is the value of the angular momentum, $A_i\,(i = 1,2,3)$ are the principal central moments of inertia relative to the axes Oz_i.

In some cases it is convenient to use, as an additional variable, along with variable θ such important characteristic as kinetic energy T, whose derivative has the form:

$$\frac{dT}{dt} = \frac{2T}{G}L_3 + G\sin\theta\left[\cos\theta\left(\frac{\sin^2\varphi}{A_1} + \frac{\cos^2\varphi}{A_2} - \frac{1}{A_3}\right)\right.$$
$$\times(L_2\cos\psi - L_1\sin\psi) \tag{1.2}$$
$$\left. + \sin\varphi\cos\varphi\left(\frac{1}{A_1} - \frac{1}{A_2}\right)(L_1\cos\psi + L_2\sin\psi)\right].$$

The satellite's center of mass moves over the Keplerian ellipse with eccentricity e and revolution frequency ω_0. The dependence of the true anomaly ν on time t is given by the relation

$$\frac{d\nu}{dt} = \frac{\omega_0(1 + e\cos\nu)^2}{\left(1 - e^2\right)^{3/2}},\quad \omega_0 = \frac{2\pi}{Q} = \sqrt{\frac{\mu\left(1 - e^2\right)^3}{l_0^3}}. \tag{1.3}$$

Here, l_0 is the focal parameter of orbit, ω_0 is the angular velocity of orbital motion, e is the orbit's eccentricity, and μ is the gravitational constant.

Projections L_i of the moment of applied forces are composed of the moment of light pressure forces L_i^c, the moment of viscous fluid forces in a cavity L_i^p and of the gravitational moment L_i^g.

Suppose that the spacecraft's surface represents the surface of rotation, the unit vector of the axis of symmetry **k** being directed along the Oz_3 axis. As is shown in [2, 5], in this case the moment of light pressure forces acting on a satellite is determined by the formula

$$\mathbf{L}^c = \left(a_c(\varepsilon_s)\,R_0^2/R^2\right)\mathbf{e}_r \times \mathbf{k},$$
$$a_c(\varepsilon_s)\frac{R_0^2}{R^2} = p_c S(\varepsilon_s)\,Z_0'(\varepsilon_s),\quad p_c = \frac{E_0}{c}\left(\frac{R_0}{R}\right)^2. \tag{1.4}$$

Here, $\mathbf{e}_r$ is the unit vector in the direction of orbit's radius vector; ε_s is the angle between directions $\mathbf{e}_r$ and $\mathbf{k}$, so that $|\mathbf{e}_r \times \mathbf{k}| = \sin\varepsilon_s$; R is the current distance from the Sun's center to satellite's center of mass; R_0 is the fixed value of R, for example, at the initial time instant; $a_c(\varepsilon_s)$ is the coefficient of the moment of light pressure forces determined by surface's properties; S is the area of a "shadow" on a plane normal to the flux; Z_0' is the distance from the center of mass to the center of pressure; p_c is the light pressure value at the distance R from Sun's center; c is the velocity of light; E_0 is the value of the light pressure energy flux at the distance R_0 from the Sun's center.

Here the projection of the gravitational moment onto the Oy_1 axis is presented, the projections onto other axes have a similar form and are obtained by rotation (shift) of indices

$$L_1^g = \frac{3\omega_0^2(1 + e\cos\nu)^3}{\left(1 - e^2\right)^3}\sum_{j=1}^{3}(\beta_2\beta_j S_{3j} - \beta_3\beta_j S_{2j}),$$
$$S_{mj} = \sum_{p=1}^{3} A_p\alpha_{jp}\alpha_{mp},\quad \beta_1 = \cos(\nu - \lambda)\cos\delta, \tag{1.5}$$
$$\beta_2 = \sin(\nu - \lambda),\quad \beta_3 = \cos(\nu - \lambda)\sin\delta.$$

Projections of the moment of forces of highly viscous fluid in a cavity L_i^p onto the axes $Oy_i\,(i = 1,2,3)$ have the following form [1]:

$$L_i^p = \frac{P}{A_1A_2A_3}$$
$$\times\left\{\boldsymbol{\omega}\mathbf{B} + \left(a_c(\cos\varepsilon_s)\frac{R_0^2}{R^2}\mathbf{C} + \frac{3\mu}{R^3}(\mathbf{D} + \mathbf{S})\right)\boldsymbol{\alpha}\right\}(i = 1,2,3), \tag{1.6}$$

$$\boldsymbol{\omega} = \begin{pmatrix} p \\ q \\ r \end{pmatrix},\quad \mathbf{B} = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix},\quad \alpha = \begin{pmatrix} \alpha_{i1} \\ \alpha_{i2} \\ \alpha_{i3} \end{pmatrix},\quad \alpha^* = \frac{1}{1 - \alpha_{33}^2},$$

$$\mathbf{C} = \begin{pmatrix} A_3\left[A_2\alpha^*(p_{\alpha1}(\gamma_{31}\alpha_{33} - \alpha_{22}\beta_1 + \alpha_{12}\beta_2) + \alpha_{32}\gamma_{33}p_{\alpha2}) - r\gamma_{31}(A_1 + A_3)\right] \\ A_3\left[A_1\alpha^*(p_{\alpha1}(\alpha_{11}\beta_2 - \alpha_{33}\gamma_{32} - \alpha_{21}\beta_1) + \alpha_{31}\gamma_{33}p_{\alpha2}) + r\gamma_{32}(A_2 + A_3)\right] \\ q\gamma_{32}A_2(A_1 - A_2 - A_3) + p\gamma_{31}A_1(A_1 - A_2 + A_3) \end{pmatrix},$$

$$\mathbf{D} = \begin{pmatrix} A_2A_3(A_3 - A_2)\{-\gamma_{31}\gamma_{33}r + \alpha^*(F_1p_{\alpha1} + M_1p_{\alpha2})\} \\ A_1A_3(A_1 - A_3)\{-\gamma_{32}\gamma_{33}r + \alpha^*(F_2p_{\alpha1} + M_2p_{\alpha2})\} \\ (A_2 - A_1)\left\{\left(\gamma_{32}^2 - \gamma_{31}^2\right)r - \alpha^*(F_3p_{\alpha1} + M_3p_{\alpha2})\right\} \end{pmatrix},$$

$$\mathbf{F} = \begin{pmatrix} \gamma_{31}\gamma_{33}\alpha_{33} + \beta_{\alpha 1}\gamma_{33} + \beta_{\alpha 2}\gamma_{32} \\ \gamma_{32}\gamma_{33}\alpha_{33} + \beta_{\alpha 3}\gamma_{33} + \beta_{\alpha 2}\gamma_{31} \\ \left(\gamma_{32}^2 - \gamma_{31}^2\right)\alpha_{33} + \beta_{\alpha 3}\gamma_{32} + \beta_{\alpha 1}\gamma_{31} \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} \gamma_{33}^2\alpha_{32} + \gamma_{32}\gamma_{33}\alpha_{33} - \gamma_{32}\beta_3 \\ \gamma_{33}^2\alpha_{31} + \gamma_{31}\gamma_{33}\alpha_{33} - \gamma_{31}\beta_3 \\ \gamma_{33}\left[\gamma_{32}\alpha_{31} + \gamma_{31}\alpha_{32}\right] \end{pmatrix},$$

$$\mathbf{S} = \begin{pmatrix} \gamma_{31}\left[\gamma_{33} r A_3\left(A_1A_2 - A_1^2 - A_2A_3 + A_3^2\right) + \gamma_{32} q A_2\left(A_1A_3 - A_1^2 - A_2A_3 + A_2^2\right)\right] \\ \gamma_{32}\left[\gamma_{31} p A_1\left(A_3A_2 - A_2^2 - A_1A_3 + A_1^2\right) + \gamma_{33} r A_3\left(A_1A_2 - A_2^2 - A_1A_3 + A_3^2\right)\right] \\ \gamma_{33}\left[\gamma_{32} q A_2\left(A_1A_3 - A_3^2 - A_1A_2 + A_2^2\right) + \gamma_{31} p A_1\left(A_2A_3 - A_3^2 - A_1A_2 + A_1^2\right)\right] \end{pmatrix},$$

$$\gamma_{3i} = \beta_1\alpha_{1i} + \beta_2\alpha_{2i} + \beta_3\alpha_{3i} \quad (i = 1,2,3),$$
$$p_{\alpha 1} = p\alpha_{31} + q\alpha_{32}, \quad p_{\alpha 2} = p\alpha_{32} - q\alpha_{31},$$
$$\beta_{\alpha i} = -\alpha_{22}\beta_1 + \alpha_{12}\beta_2, \quad \beta_{\alpha 2} = -\alpha_{23}\beta_1 + \alpha_{13}\beta_2,$$
$$\beta_{\alpha 3} = -\alpha_{21}\beta_1 + \alpha_{11}\beta_2,$$
$$B_1 = \left[\omega_2^2 A_2(A_1 - A_2)(A_2 - A_3 + A_1)\right.$$
$$\left. + \omega_3^2 A_3(A_1 - A_3)(A_3 - A_2 + A_1)\right]\alpha_{i1},$$

B_2 and B_3 have a similar form and are obtained by rotation (shift) of indices.

Here, α_{ij} are direction cosines between coordinate systems $Oy_i\,(i = 1,2,3)$ and $Oz_i\,(i = 1,2,3)$, p, q, r are projections of the absolute angular velocity vector $\boldsymbol{\omega}$ of the satellite relative to the $Ox_1x_2x_3$ coordinate system onto the $Oz_i\,(i = 1,2,3)$ axes.

Quantity $\tilde{P}$ is a tensor depending on the cavity shape only, it characterizes the dissipative moment of forces, caused by viscous fluid, in the quasistatic approximation [1]. For the sake of simplicity, in Eqs. (1.6) the so-called scalar tensor is considered, which is determined by a single scalar quantity $P > 0$. The components of this tensor are $\tilde{P}_{ij} = P\delta_{ij}$, where δ_{ij} are Kronecker's symbols (tensor $\tilde{P}$ has such a form, for example, in the case of spherical cavity). If the cavity's shape essentially differs from spherical one, the determination of tensor components is associated with considerable computation difficulties.

The dynamically asymmetric satellite is considered, whose moments of inertia, for certainty, satisfy the inequality $A_1 > A_2 > A_3$, under an assumption that the angular velocity ω of satellite motion relative to the center of mass is essentially larger than the angular velocity of orbital motion ω_0, i.e., $(\varepsilon = \omega_0/\omega \sim A_1\omega_0/G \ll 1$. In this case the kinetic energy of body rotation is large as compared to moments of disturbing forces.

It is supposed in the paper, that the cavity is filled with the high-viscosity fluid, i.e., $\vartheta \gg 1\left(\vartheta^{-1} \sim \varepsilon^2\right)$, the cavity's shape is spherical; then [1]

$$\tilde{P} = P\,\mathrm{diag}(1,1,1), \quad P = 8\pi\sigma b_0^7/525\vartheta. \qquad (1.7)$$

Here σ and ϑ are the density and kinematical coefficient of viscosity of fluid in a cavity, respectively, b_0 is the cavity radius.

We suppose [2] that by virtue of symmetry the corresponding function has a form $a_c = a_c(\cos\varepsilon_s)$, and we approximate it by trigonometric polynomials in powers of $\cos\varepsilon_s$. Let us present the function $a_c(\cos\varepsilon_s)$ as $a_c = a_0 + a_1\cos\varepsilon_s + \ldots$. Consider now the second term of the expansion, when $a_c(\cos\varepsilon_s) = a_1\cos\varepsilon_s$ provided that $a_1 \sim \varepsilon$.

With accounting for the assumptions considered above one can see that the second term (with coefficient $a_c(\cos\varepsilon_s)$) in the formula for projection of the moment of forces of viscous fluid in a cavity (1.6) has the order of ε^3. The gravitation constant μ is proportional to the squared angular velocity of orbital motion ω_0, i.e., $\mu \sim \varepsilon^2$. Therefore, to an accuracy of quantities of the second order of smallness ($P \sim \varepsilon^2$), and the projections of the moment of forces of viscous fluid in a cavity have the form:

$$\begin{aligned} L_i^p = \frac{P}{A_1A_2A_3}\Big\{ & p\left[q^2 A_2(A_1 - A_2)(A_2 - A_3 + A_1)\right. \\ & \left. + r^2 A_3(A_1 - A_3)(A_3 - A_2 + A_1)\right]\alpha_{i1} \\ & + q\left[r^2 A_3(A_2 - A_3)(A_3 - A_1 + A_2)\right. \\ & \left. + p^2 A_1(A_1 - A_2)(A_3 - A_1 - A_2)\right]\alpha_{i2} \\ & + r\left[p^2 A_1(A_3 - A_1)(A_1 - A_2 + A_3) + q^2 A_2(A_3 - A_2)\right. \\ & \left. \times (A_2 - A_1 + A_3)\right]\alpha_{i3}\Big\} \quad (i = 1,2,3). \end{aligned} \qquad (1.8)$$

The problem is formulated to study the evolution of satellite rotations over an asymptotically large time interval $t \sim \varepsilon^{-2}$, on which the motion parameters essentially change.

2. MODIFIED PROCEDURE OF THE METHOD OF AVERAGING

For the considered problem of solving system (1.1) – (1.3) at small ε over the time interval $t \sim \varepsilon^{-2}$ we apply the modified scheme of the method of averaging [3, 16, 17]. Let us consider the undisturbed motion ($\varepsilon = 0$), when the moments of applied forces are zero. In this case the solid body rotation represents the Euler–Poinsot motion. Quantities G, δ, λ, T, and ν are converted into constants, and φ, ψ, and θ are some functions of time t. Slow variables in the disturbed motion are represented by G, δ, λ, T, and ν, while the fast variables are the Euler angles φ, ψ, and θ.

Consider now the motion under the condition $2TA_1 \geq G^2 > 2TA_2$, which corresponds to the angular momentum vector trajectories enveloping the axis of the largest moment of inertia A_1.. We introduce the following quantity

$$k^2 = \frac{(A_2 - A_3)(2TA_1 - G^2)}{(A_1 - A_2)(G^2 - 2TA_3)} \quad (0 \leq k^2 \leq 1), \qquad (2.1)$$

which represents, in the undisturbed motion, a constant—the modulus of elliptic functions describing this motion.

For constructing the averaged system of the first approximation we substitute the solution to the undisturbed Euler–Poinsot motion into the right-hand sides of equations of motion (1.1), (1.2) and perform averaging over the variable ψ, and then over time t taking into account the dependence of φ, θ on t according to the scheme proposed in [3] for the non-resonant case. Here, the former designations are retained for slow variables δ, λ, G, and T. As a result, we get

$$\frac{dG}{dt} = 0, \quad \frac{d\delta}{dt} = -a_1 R_0^2 \left(2GR^2\right)^{-1}$$
$$\times H \sin\delta \sin 2(\lambda - \nu) - \frac{3\omega_0^2 (1 + e\cos\nu)^3}{2G\left(1 - e^2\right)^3} \beta_2 \beta_3 N^*,$$

$$\frac{d\lambda}{dt} = -a_1 R_0^2 \left(GR^2\right)^{-1}$$
$$\times H \cos\delta \cos^2(\lambda - \nu) + \frac{3\omega_0^2 (1 + e\cos\nu)^3}{2G\left(1 - e^2\right)^3 \sin\delta} \beta_1 \beta_3 N^*, \qquad (2.2)$$

$$\frac{dT}{dt} = -\frac{4PT^2 (A_1 - A_3)(A_1 - A_2)(A_2 - A_3)}{3A_1^2 A_2^2 A_3^2 S^2(k)}$$
$$\times\left\{A_2 (A_1 - A_3)(A_1 + A_3 - A_2)\left[k^2 V(k) - W(k)\right]\right.$$
$$+ A_1 (A_2 - A_3)(A_3 + A_2 - A_1)\left[\left(k^2 - 2\right) W(k) + k^2\right]$$
$$\left. + A_3 (A_1 - A_2)(A_1 + A_2 - A_3)\left[\left(1 - 2k^2\right) W(k) + k^2\right]\right\},$$

$$S(k) = A_2 - A_3 + (A_1 - A_2) k^2, \quad V(k) = 1 + \frac{E(k)}{K(k)},$$
$$W(k) = 1 - \frac{E(k)}{K(k)},$$

$$H = \frac{1}{2}\left[3a^2 \frac{E(k)}{K(k)} - 1\right] \quad \text{at} \quad 2TA_2 - G^2 > 0,$$

$$H = \frac{1}{2}\left\{\frac{3a^2}{k^2}\left[k^2 - 1 + \frac{E(k)}{K(k)}\right] - 1\right\} \quad \text{at} \quad 2TA_2 - G^2 < 0,$$

$$a^2 = \frac{\sigma + h}{1 + \sigma}, \quad \sigma = \frac{A_3}{A_1}\frac{A_1 - A_2}{A_2 - A_3}, \quad h = \left(\frac{2T}{G^2} - \frac{1}{A_2}\right)\frac{A_2 A_3}{A_2 - A_3},$$

$$N^* = A_2 + A_3 - 2A_1$$
$$+ 3\left(\frac{2A_1 T}{G^2} - 1\right)\left[A_3 + (A_2 - A_3)\frac{K(k) - E(k)}{K(k) k^2}\right].$$

Here, $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind, respectively [18]. According to the first equation of (2.2), the angular momentum of a satellite remains constant and equal to G_0. Differentiating the expression for k^2 (2.1) and using equations for kinetic energy (2.2), we obtain the differential equation, which does not depend on other variables [1, 11]

$$\frac{dk^2}{d\xi} = (1 - \chi)\left(1 - k^2\right) - \left[(1 - \chi) + (1 + \chi) k^2\right]\frac{E(k)}{K(k)},$$

$$\chi = \frac{3A_2\left[\left(A_1^2 + A_3^2\right) - A_2 (A_1 + A_3)\right]}{(A_1 - A_3)\left[A_2 (A_1 + A_3 - A_2) + 2A_1 A_3\right]}, \qquad (2.3)$$

$$\xi = (t - t_*)/N,$$

$$N = \frac{3A_1^2 A_2^2 A_3^2}{PG_0^2 (A_1 - A_3)\left[A_2 (A_1 + A_3 - A_2) + 2A_1 A_3\right]} \sim \varepsilon^{-2}.$$

Here t_* is a constant. The equality $2TA_2 = G^2$, corresponds to the value of $k^2 = 1$, which corresponds to a separatrix for the Euler–Poinsot motion. Equation (2.3) describes the averaged motion of a tip of the angular momentum vector **G** over the sphere of constant radius G_0.

3. ANALYSIS OF AVERAGED PROPER ROTATION OF A SATELLITE

It follows from equations of motion (2.2) that, under an effect of the moment of forces of viscous fluid in a cavity, body's kinetic energy T evolves within the limits from the rotation around axis A_3 (unstable motion) to rotation around the axis A_1 (stable motion). Changes of angles λ and δ depend both on the action of external moments of light pressure and gravitational forces, and on the action of the internal moment of forces of viscous fluid in the cavity. The expression in curly brackets of the right-hand side of Eq. (2.2) for T

is positive (for $A_1 > A_2 > A_3$), because the inequalities $(1-k^2)K \le E \le K$. are valid. Therefore, $dT/dt < 0$ since $T > 0$, i.e., the variable T strictly decreases for any $k^2 \in [0,1]$.

Let us consider now the system consisting of the fourth equation of system (2.2) and Eq. (2.3). We make the equation for the kinetic energy change dimensionless, considering quantity N (2.3) and the moment of inertia A_1 to be characteristic quantities of the problem. We have

$$\frac{d\tilde{T}}{d\xi} = -\frac{2(\tilde{T})^2(A_1-A_2)(A_2-A_3)}{A_1[A_2(A_1+A_3-A_2)+2A_1A_3]S^2(k^2)}\left\{A_2(A_1-A_3)(A_1+A_3-A_2)\left[k^2V(k)-W(k)\right]\right. \tag{3.1}$$
$$\left.+A_1(A_2-A_3)(A_3+A_2-A_1)\left[\left(k^2-2\right)W(k)+k^2\right]+A_3(A_1-A_2)(A_1+A_2-A_3)\left[\left(1-2k^2\right)W(k)+k^2\right]\right\},$$

where $\tilde{T} = \dfrac{2A_1T}{G_0^2}$, ξ is determined according to (2.3). This equality is valid for $\xi > 0$, i.e., for the case $2TA_1 \ge G^2 \ge 2TA_2$.

The numerical calculation was performed for the values of moments of inertia $A_1 = 8$, $A_2 = 5, 6, 7$, $A_3 = 4$; $k^2(0) = = 0.99999$, $G(0) = 1$. The initial value of kinetic energy was found from the equality

$$T = \frac{G_0^2}{2}\frac{A_2-A_3+(A_1-A_2)k^2(0)}{A_1(A_2-A_3)+A_3(A_1-A_2)k^2(0)}. \tag{3.2}$$

In the dimensionless form we have

$$\tilde{T} = \frac{A_1\left(A_2-A_3+(A_1-A_2)k^2(0)\right)}{A_1(A_2-A_3)+A_3(A_1-A_2)k^2(0)}.$$

We have considered also the case of $\xi < 0$, which corresponds to the case $2TA_2 > G^2 \ge 2TA_3$. Equation (2.3) is written in the following form:

$$\frac{d\tilde{T}}{d\xi} = \frac{2(\tilde{T})^2(A_3-A_2)(A_2-A_1)}{A_3[A_2(A_1+A_3-A_2)+2A_1A_3]S^2(k)}$$
$$\times\left\{A_2(A_1-A_3)(A_1+A_3-A_2)\left[k^2V(k)-W(k)\right]\right.$$
$$+A_1(A_2-A_3)(A_3+A_2-A_1)\left[\left(k^2-2\right)W(k)+k^2\right]$$
$$\left.+A_3(A_1-A_2)(A_1+A_2-A_3)\left[\left(1-2k^2\right)W(k)+k^2\right]\right\}$$

with the initial condition

$$\tilde{T} = \frac{A_3\left(A_2-A_3+(A_1-A_2)k^2(0)\right)}{A_1(A_2-A_3)+A_3(A_1-A_2)k^2(0)}.$$

In this case the numerical calculation was performed for the values of moments of inertia $A_1 = 4$, $A_2 = 5, 6, 7$, $A_3 = 8$. The plots of kinetic energy change have the form presented in Fig. 1.

The plots of kinetic energy change have such a form in the case, when the satellite with a cavity rotates only under the action of the gravitational moment [14], or only under the action of light pressure [15], because the evolution of quantity T is influenced only by the moment of forces of viscous fluid fully filling the cavity.

Curves *1*, *2*, *3* correspond to various values of $A_2 = 5, 6, 7$. The value $\tilde{T} = 2$ corresponds to rotation around the axis A_3 (unstable motion), $\tilde{T} = 1$ represents rotation around the axis A_1 (stable motion). For $\xi = 0$ (passage through the separatrix) the curves have a horizontal tangent line (inflexion points). Similar plots of kinetic energy change can be obtained by recalculation from formula (2.1) for the dimensionless kinetic energy

$$\tilde{T} = \frac{A_1S(k)}{A_1(A_2-A_3)+k^2A_3(A_1-A_2)}.$$

It is seen from this expression that for $k^2 \to 0$ we have $\tilde{T} \to 1$. Similarly, for the case of rotation around the axis A_3 one can demonstrate that $\tilde{T} \to 2$.

4. ANGULAR MOMENTUM VECTOR ORIENTATION

Let us consider the system consisting of the equations for λ and δ of system (2.2). As is known, $R = = l_0/(1+e\cos\nu)$, and the focal parameter of orbit is determined by the equality $l_0 = \mu^{1/3}\left(1-e^2\right)\omega_0^{-2/3}$. Then the first two equations of (2.2) take on the form:

$$\frac{d\delta}{dt} = -\frac{\omega_0^{4/3}(1+e\cos\nu)^2}{2G\left(1-e^2\right)^2}\left[\frac{a_1R_0^2}{\mu^{2/3}}H\sin\delta\sin 2(\lambda-\nu)\right. \tag{4.1}$$
$$\left.+\frac{3(1+e\cos\nu)\omega_0^{2/3}}{1-e^2}\beta_2\beta_3N^*\right],$$
$$\frac{d\lambda}{dt} = -\frac{\omega_0^{4/3}(1+e\cos\nu)^2}{G\left(1-e^2\right)^2}$$
$$\times\left[\frac{a_1R_0^2}{\mu^{2/3}}H\cos\delta\cos^2(\lambda-\nu)-\frac{\omega_0^{2/3}(1+e\cos\nu)}{2\left(1-e^2\right)\sin\delta}\beta_1\beta_3N^*\right].$$

We make the equation of angular momentum change (2.2), equations for a true anomaly (1.3) and k^2 (2.3), and the equations of system (4.1) dimensionless. Characteristic parameters of the problem are as follows: G_0 is the angular momentum of a satellite for $t = 0$, and Ω_0 is the value of angular velocity ω of satellite motion relative to the center of mass at the initial time instant. Dimensionless quantities are determined by the formulas $\tilde{t} = \Omega_0 t$, $\tilde{G} = G/G_0$, $\tilde{A}_i = A_i\Omega_0/G_0$, $\tilde{L}_i = L_i/(G_0\Omega_0)$, $\tilde{T} = T/(G_0\Omega_0)$, $\varepsilon^2\tilde{P} = P\Omega_0^2/G_0$.

We introduce the designation

$$\Gamma = \frac{a_1 R_0^2 \Omega_0}{G_0 \mu^{2/3} \omega_0^{2/3}} \quad (4.2)$$

and call this quantity the normalized coefficient of a moment of light pressure forces.

After making equations dimensionless we have the system of equations of motion in the form:

$$\frac{d\delta}{d\tilde{t}} = -\varepsilon^2 \frac{(1+e\cos\nu)^2}{2\tilde{G}(1-e^2)^2} \times \left(\Gamma \tilde{H} \sin\delta \sin 2(\lambda-\nu) + \frac{3(1+e\cos\nu)}{2(1-e^2)}\beta_2\beta_3\tilde{N}^*\right),$$

$$\frac{d\lambda}{d\tilde{t}} = \varepsilon^2 \frac{(1+e\cos\nu)^2}{\tilde{G}(1-e^2)^2} \times \left(\frac{3(1+e\cos\nu)}{2(1-e^2)\sin\delta}\beta_1\beta_3\tilde{N}^* - \Gamma\tilde{H}\cos\delta\cos^2(\lambda-\nu)\right),$$

$$\frac{d\nu}{d\tilde{t}} = \varepsilon\frac{(1+e\cos\nu)^2}{(1-e^2)^{3/2}}, \quad \frac{d\tilde{G}}{d\tilde{t}} = 0, \quad (4.3)$$

$$\frac{dk^2}{d\tilde{t}} = \varepsilon^2\frac{1}{\tilde{N}} \times \left\{(1-\chi)(1-k^2) - \left[(1-\chi)+(1+\chi)k^2\right]\frac{E(k)}{K(k)}\right\},$$

$$\tilde{N} = \frac{3\tilde{A}_1^2\tilde{A}_2^2\tilde{A}_3^2}{\tilde{P}(\tilde{A}_1-\tilde{A}_3)[\tilde{A}_2(\tilde{A}_1+\tilde{A}_3-\tilde{A}_2)+2\tilde{A}_1\tilde{A}_3]},$$

$$\tilde{H} = \frac{1}{2}\left[3\tilde{a}^2\frac{E(k)}{K(k)} - 1\right] \quad \text{at} \quad 2\tilde{T}\tilde{A}_2 - \tilde{G}^2 > 0,$$

$$\tilde{H} = \frac{1}{2}\left\{\frac{3\tilde{a}^2}{k^2}\left[k^2 - W(k)\right] - 1\right\} \quad \text{at} \quad 2\tilde{T}\tilde{A}_2 - \tilde{G}^2 < 0,$$

$$\tilde{a}^2 = \frac{\tilde{\sigma}+\tilde{h}}{1+\tilde{\sigma}}, \quad \tilde{\sigma} = \frac{\tilde{A}_3(\tilde{A}_1-\tilde{A}_2)}{\tilde{A}_1(\tilde{A}_2-\tilde{A}_3)}, \quad \tilde{h} = \left(\frac{2\tilde{T}}{\tilde{G}^2} - \frac{1}{\tilde{A}_2}\right)\frac{\tilde{A}_2\tilde{A}_3}{\tilde{A}_2-\tilde{A}_3},$$

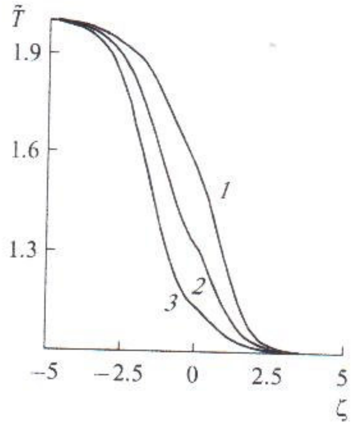

Fig. 1.

$$\tilde{N}^* = \tilde{A}_2 + \tilde{A}_3 - 2\tilde{A}_1 + 3\left(\frac{2\tilde{A}_1\tilde{T}}{\tilde{G}^2} - 1\right)\left[\tilde{A}_3 + (\tilde{A}_2-\tilde{A}_3)\frac{K(k)-E(k)}{K(k)k^2}\right],$$

$$\frac{d\tilde{T}}{d\tilde{t}} = -\varepsilon^2\frac{4\tilde{P}\tilde{T}^2(\tilde{A}_1-\tilde{A}_3)(\tilde{A}_1-\tilde{A}_2)(\tilde{A}_2-\tilde{A}_3)}{3\tilde{A}_1^2\tilde{A}_2^2\tilde{A}_3^2\left[\tilde{A}_2-\tilde{A}_3+(\tilde{A}_1-\tilde{A}_2)k^2\right]^2}$$

$$\times\left\{\tilde{A}_2(\tilde{A}_1-\tilde{A}_3)(\tilde{A}_1+\tilde{A}_3-\tilde{A}_2)\left[k^2V(k)-W(k)\right]\right.$$

$$+\tilde{A}_1(\tilde{A}_2-\tilde{A}_3)(\tilde{A}_3+\tilde{A}_2-\tilde{A}_1)\left[(k^2-2)W(k)+k^2\right]$$

$$\left.+\tilde{A}_3(\tilde{A}_1-\tilde{A}_2)(\tilde{A}_1+\tilde{A}_2-\tilde{A}_3)\left[(1-2k^2)W(k)+k^2\right]\right\}.$$

The first three equations for λ, δ, and ν of system (4.3) can be written as follows:

$$\frac{d\delta}{d\tilde{t}} = \varepsilon^2\Delta(\nu,\delta,\lambda), \quad \frac{d\lambda}{d\tilde{t}} = \varepsilon^2\Lambda(\nu,\delta,\lambda),$$
$$\frac{d\nu}{d\tilde{t}} = \varepsilon\frac{(1+e\cos\nu)^2}{h(e)}, \quad h(e) = (1-e^2)^{3/2}. \quad (4.4)$$

Here, Δ, Λ are coefficients in the right-hand sides of the first and second equations of (4.3), δ, λ are slow variables, and ν is a semi-slow one.

We have obtained the system of special type, whose solution is found by a modified method of averaging according to the following scheme [17]:

$$\frac{d\delta}{d\tilde{t}} = \varepsilon^2\frac{h(e)}{2\pi}\int_0^{2\pi}\frac{\Delta(\lambda,\delta,\nu)}{(1+e\cos\nu)^2}d\nu,$$

$$\frac{d\lambda}{d\tilde{t}} = \varepsilon^2\frac{h(e)}{2\pi}\int_0^{2\pi}\frac{\Lambda(\lambda,\delta,\nu)}{(1+e\cos\nu)^2}d\nu.$$

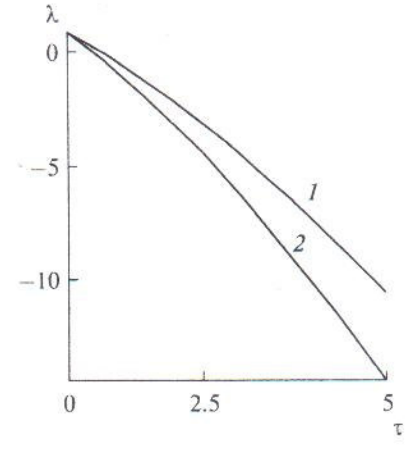

Fig. 2.

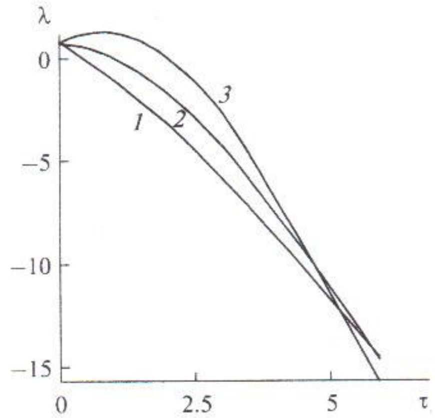

Fig. 3.

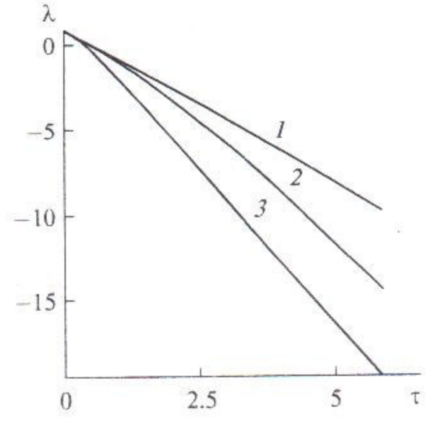

Fig. 4.

After averaging we get

$$\frac{d\delta}{d\tilde{t}} = 0, \quad \frac{d\lambda}{d\tilde{t}} = \varepsilon^2 \left(\frac{3\tilde{N}^*}{h(e)} - \frac{2\Gamma \tilde{H}}{\left(1-e^2\right)^{1/2}} \right) \frac{\cos\delta}{4\tilde{G}}.$$

The system was integrated for slow time $\tau = \varepsilon^2 \tilde{t}$. The numerical calculation was performed at the initial conditions $\tilde{G}(0) = 1$; $k^2(0) = 0.99$; $\delta(0) = 0.785$ rad; and $\lambda(0) = 0.785$ rad. The orbits with the following eccentricities were considered: $e = 0$ (circular orbit) and $e = 0.421$ (highly elliptical orbit). For the dimensionless time τ we have the following picture of changes in the angle of orientation of the angular momentum vector, presented in Fig. 2. Curves *1* and 2 correspond to the circular and highly elliptical orbits, respectively.

Figure 3 presents the plots of change of the same angle for various values of satellite's moments of inertia. Curves *2*, and *3* correspond to various values of $\tilde{A}_2 = 7, 6, 5$ for constant values $\tilde{A}_1 = 8$, $\tilde{A}_3 = 4$. It is seen in Fig. 3 that the character of change of angle λ for close values of moments of inertia $\tilde{A}_1$ and $\tilde{A}_2$ is almost linear. As the value of the moment of inertia $\tilde{A}_2$ decreases, the curvature of function grows, and in this case the function ceases to be monotonous.

The character of change of angle λ has the same form, as in the problem of motion of a satellite with the cavity filled with viscous fluid in the gravitational field [14].

In the case of motion of a satellite with the cavity filled with viscous fluid under the action of the moment of light pressure forces [15], the character of change of angle λ is almost linear, and, as the value of the dimensionless moment of inertia $\tilde{A}_2$ increases, the function increases more rapidly.

One can also analyze the changes in the character of function $\lambda(\tau)$ for various values of the dimensionless quantity $\tilde{P}$. Curves *1*, *2*, and *3* in Fig. 4 correspond to various values of $\tilde{P}(0) = 10$, 100, 1000. It is seen that the character of variation of the angle has almost linear form.

According to the numerical calculation, it is shown that for the asymmetric satellite with a cavity filled with viscous fluid, which moves under the action of the moment of light pressure forces in the gravitational field, the angular momentum vector **G** remains to be a constant quantity directed at constant angle δ to the vertical of the orbital plane. In this case, the tip of vector **G** moves over the sphere of radius G_0 clockwise, and the kinetic energy decreases down to the dimensionless value of 1 corresponding to stable motion of a satellite around the axis A_1. The same direction of motion of the tip of the angular momentum vector is characteristic for the problems on motion of a satellite with a cavity under the action of the moment of forces of gravitational attraction [14] and of the moment of light pressure forces [15].

5. EXTREME CASE OF ROTATION CLOSE TO AXIAL ONE

We consider the motion of a body for small $k^2 \ll 1$, corresponding to solid body motions close to rotations around the A_1 axis. In this case, the right-hand side of Eq. (2.2) can be simplified using the expansions of complete elliptic integrals into series in terms of k^2 [18]. Then, Eq. (2.2) can be integrated, and the asymptotical solution is written in the form

$$k^2 = C_1 \exp\left[-\frac{(3+\chi)\xi}{2}\right] \text{ at } \xi > 0,$$
$$k^2 = C_1 \exp\left[\frac{(3-\chi)\xi}{2}\right] \text{ at } \xi < 0, \quad (5.1)$$
$$C_1 = \text{const}, \quad 0 \le C_1 \le 1.$$

The change of kinetic energy can be roughly obtained qualitatively, following paper [1], by simple conversion from relation (2.1), using the found solution for small k^2 (5.1). We have

$$T = \frac{G^2}{2A_1} + \frac{G^2(A_1 - A_3)(A_1 - A_2)}{2A_1^2(A_2 - A_3)} \times C_1 \exp\left[-\frac{(3+\chi)\xi}{2}\right] \text{ at } \xi > 0, \quad (5.2)$$
$$T = \frac{G^2}{2A_3} + \frac{G^2(A_3 - A_1)(A_3 - A_2)}{2A_3^2(A_2 - A_1)} C_1 \exp\left[\frac{(3-\chi)\xi}{2}\right] \text{ at } \xi < 0.$$

For the dimensionless value of kinetic energy equalities (5.2) take on the form

$$T^* = 1 + \frac{(A_1 - A_3)(A_1 - A_2)}{A_1(A_2 - A_3)} C_1 \exp\left[-\frac{(3+\chi)\xi}{2}\right] \text{ at } \xi > 0,$$
$$T^* = \frac{A_1}{A_3} + \frac{A_1(A_3 - A_1)(A_3 - A_2)}{A_3^2(A_2 - A_1)} C_1 \exp\left[\frac{(3-\chi)\xi}{2}\right] \text{ at } \xi < 0. \quad (5.3)$$

The integration constant C_1 is found roughly from the condition of equality of kinetic energy by formulas (5.3) at $\xi = 0$. We have

$$C_1 = \frac{A_1 A_3 (A_2 - A_3)(A_1 - A_2)}{A_3^2(A_1 - A_2)^2 + A_1^2(A_2 - A_3)^2}. \quad (5.4)$$

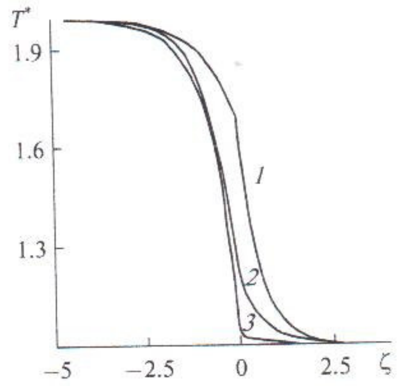

Fig. 5.

The plots of changes in dimensionless kinetic energy T^* in the case of small k^2 have the form presented in Fig. 5. Curves *1*, *2*, and *3* correspond to various values of $A_2 = 5, 6, 7$, with constant values of $A_1 = 8$, $A_3 = 4$ for $\xi > 0$ and $A_1 = 4$, $A_3 = 8$ for $\xi < 0$. As is seen in the figure, the character of function $T^* = T^*(\xi)$ is the same, as for $0 \le k^2 \le 1$, and, also, the asymptotic values of T^* retain their values at positive and negative dimensionless times.

The asymptotical expression for the modulus of elliptic functions can be presented as a function of the dimensionless time τ

$$k^2 = k_0^2 \exp[-\rho\tau],$$
$$\rho = \frac{\tilde{P}}{\tilde{A}_1^2 \tilde{A}_2^2 \tilde{A}_3^2}\left[\tilde{A}_1\tilde{A}_2(\tilde{A}_1 - \tilde{A}_2) + \tilde{A}_1\tilde{A}_3(\tilde{A}_1 - \tilde{A}_3) + \tilde{A}_2\tilde{A}_3\right]. \quad (5.5)$$

Let us consider the differential equation for changes in angle τ (4.4) in the dimensionless time τ for small k^2 with accounting for (5.5). The right-hand side of the equation includes the non-constant quantity $\tilde{H}$. At $2\tilde{T}\tilde{A}_2 - \tilde{G}^2 < 0$ function $\tilde{H}(\tau)$ with allowance for the terms of the second order of smallness has the form:

$$\tilde{H} = \frac{1}{2}\left\{\frac{3\tilde{A}_3(\tilde{A}_1 - \tilde{A}_2)}{2\tilde{A}_1(\tilde{A}_2 - \tilde{A}_3)} k_0^2 \exp[-\rho\tau] - 1\right\}.$$

It is seen that for $\tau \to \infty$ quantity $\tilde{H} \to -0.5$.

The asymptotic expression of the kinetic energy can be represented as a function of dimensionless time τ

$$\tilde{T} = \frac{\tilde{G}^2}{2\tilde{A}_1} + \frac{\tilde{G}^2(\tilde{A}_1 - \tilde{A}_3)(\tilde{A}_1 - \tilde{A}_2)}{2\tilde{A}_1^2(\tilde{A}_2 - \tilde{A}_3)} k_0^2 \exp[-\rho\tau].$$

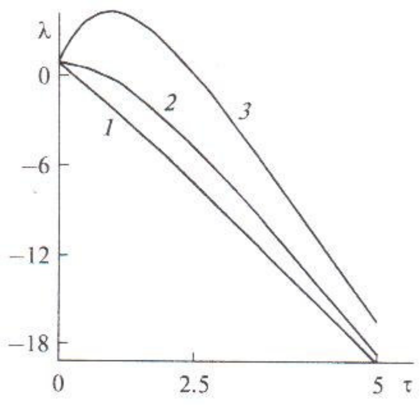

Fig. 6.

Substitute the obtained expression for $\tilde{H}$ and $\tilde{T}$ into the equation for changes in angle λ, we integrate the result and find

$$\lambda = \frac{\cos\delta}{4\tilde{G}_0(1-e^2)^{1/2}}\left\{\frac{3(\tilde{A}_1 - \tilde{A}_2)k_0^2}{\tilde{A}_1(\tilde{A}_2 - \tilde{A}_3)\rho}\right.$$

$$\times\left(\Gamma\tilde{A}_3 - \frac{3(\tilde{A}_2 + \tilde{A}_3)(\tilde{A}_1 - \tilde{A}_3)}{1-e^2}\right)(\exp[-\rho\tau] - 1)$$

$$\left. + \left(\frac{3}{1-e^2}(\tilde{A}_2 + \tilde{A}_3 - 2\tilde{A}_1) + \Gamma\right)\tau\right\} + \lambda_0,$$

where λ_0, k_0^2 are determined from the initial conditions. The plot of this function $\lambda = \lambda(\tau)$ for $k^2 \ll 1$ has the form presented in Fig. 6.

Curves *1*, *2*, and *3* correspond to various values of $\tilde{A}_2 = 7, 6, 5$, for constant values of $\tilde{A}_1 = 8$, $\tilde{A}_3 = 4$ and for the initial value of angle $\lambda(0) = 0.785$ rad. As is seen in the figure, the character of curves is similar to functions $\lambda = \lambda(\tau)$ for arbitrary k_2.

The change of angle λ, for small k_2 has approximately the same form, as in the case of motion of a satellite with the cavity, filled with viscous fluid, in the gravitational field [14]. In our problem, however, the decrease of the angle of orientation occurs slightly more rapidly.

During the motion of a satellite with viscous fluid under the action of the moment of light pressure forces [15], angle λ increases, as in the case of satellite motion under the action of the moment of light pressure forces in the resistant medium [8].

6. THE MOTION OF A DYNAMICALLY SYMMETRIC SATELLITE

Let us consider the motion of a dynamically symmetric satellite ($A_1 = A_2$), whose moments of inertia, for certainty, satisfy the inequality $A_1 > A_3$. We write the equations of body motion relative to the center of mass in the form [2]

$$\frac{dG}{dt} = L_3,\ \frac{d\delta}{dt} = \frac{L_1}{G},\ \frac{d\lambda}{dt} = \frac{L_2}{G\sin\delta},$$
$$\frac{d\theta}{dt} = \frac{L_2\cos\psi - L_1\sin\psi}{G},$$
$$\frac{d\varphi}{dt} = G\cos\theta\left(\frac{1}{A_3} - \frac{1}{A_1}\right) + \frac{L_1\cos\psi + L_2\sin\psi}{G\sin\theta}, \tag{6.1}$$
$$\frac{d\psi}{dt} = \frac{G}{A_1} - \frac{L_1\cos\psi + L_2\sin\psi}{G}\mathrm{ctg}\theta - \frac{L_2}{G}\mathrm{ctg}\delta.$$

Projections of the moment of forces of viscous fluid in the cavity L_i^p onto the axes Oy_i $(i = 1, 2, 3)$ for $A_1 = A_2$ have the form:

$$L_i^p = \frac{P}{A_1A_2}(A_1 - A_3)$$
$$\times\left\{pr^2A_3\alpha_{i1} + qr^2A_3\alpha_{i2} - rA_1\left[p^2 + q^2\right]\alpha_{i3}\right\}\ (i = 1, 2, 3). \tag{6.2}$$

To solve the problem we apply the method of averaging [15]. In the case of undisturbed Euler–Poinsot motion, when the ellipsoid of inertia represents the ellipsoid of rotation, φ and ψ are linear functions, and angle θ is the constant quantity [19]. For the disturbed motion angles φ and ψ are fast variables, and angle θ is slow one. We perform averaging of the systems of equations for slow variables G, δ, λ, and θ over fast variables: first over ψ and then over φ.

After averaging over fast variables φ and ψ we have the equations in dimensionless quantities

$$\frac{dG^*}{dt^*} = 0,\ \frac{d\theta}{dt^*} = \varepsilon^2\Gamma_1(A_1^* - A_3^*)\sin\theta\cos\theta, \tag{6.3}$$

$$\frac{d\delta}{dt^*} = -\varepsilon^2\frac{(1+e\cos\nu)^2}{2(1-e^2)^2}\left(1 - \frac{3}{2}\sin^2\theta\right)$$
$$\times\sin\delta\sin 2(\lambda - \nu)\left\{\Gamma - \frac{3(1+e\cos\nu)}{G^*(1-e^2)}\left(A_1^* - A_3^*\right)\right\},$$

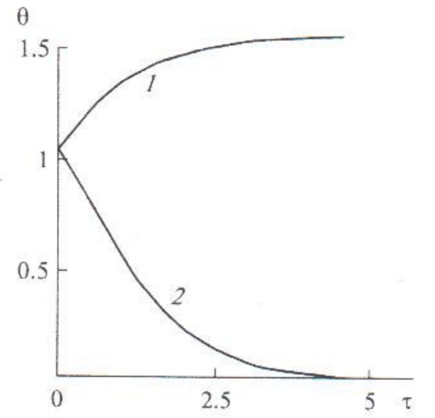

Fig. 7.

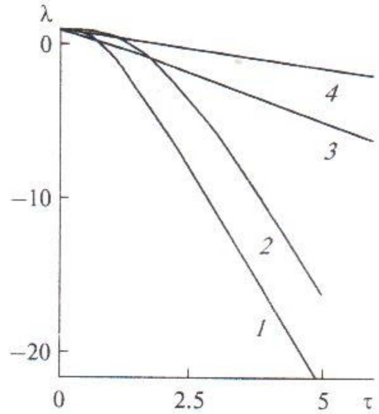

Fig. 8

$$\frac{d\lambda}{dt^*} = \varepsilon^2 \frac{(1+e\cos\nu)^2}{(1-e^2)^2}\left(1-\frac{3}{2}\sin^2\theta\right)$$
$$\times \cos\delta\cos^2(\lambda-\nu)\left\{\frac{3(1+e\cos\nu)}{G^*(1-e^2)}(A_1^*-A_3^*)-\Gamma\right\}.$$

Here, the dimensionless quantities are determined by equalities $t^* = \Omega_0 t$, $A_i^* = A_i\Omega_0/G_0$, $\varepsilon^2\vartheta^* = \vartheta/\Omega_0 a^2$, where Ω_0 is the angular velocity of satellite motion relative to the center of mass at the initial time instant.

The following designations are introduced: Γ according to (4.1) and $\Gamma_1 = \frac{8\pi a^5\rho G_0^3}{525\nu^* A_1^3 A_3 \Omega_0^3}$, where μ is the gravitational constant. We call quantity Γ_1 the normalized coefficient of the moment of forces of viscous fluid in a cavity.

Let us investigate the solution to system (6.3) for small ε on the time interval $\tau = \varepsilon^2 t^*$. It is seen from the first equation of system (6.3), that the angular momentum is a constant quantity. Integrating the second equation of system (6.3) for the nutation angle, we obtain

$$\mathrm{tg}\theta = \mathrm{tg}\theta_0 \exp\left[\Gamma_1\left(A_1^*-A_3^*\right)\tau\right]. \qquad (6.4)$$

The plot of function $\theta = \theta(\tau)$ has the form presented in Fig. 7. The calculation was carried out at the initial condition $\theta(0) = \pi/3$ рад. rad. Curve *1* corresponds to the case of $A_1^* > A_3^*$ (the satellite is "oblate" in the axis of inertia A_3), and curve *2* corresponds to the case of $A_1^* < A_3^*$ (the satellite is "elongated" in the axis of inertia A_3).

The last two equations of (6.3) and the equation for the true anomaly (1.3) in the dimensionless time τ can be written as

$$\frac{d\delta}{dt^*} = \varepsilon^2\Delta(\nu,\delta,\lambda), \quad \frac{d\lambda}{dt^*} = \varepsilon^2\Lambda(\nu,\delta,\lambda),$$
$$\frac{d\nu}{dt^*} = \frac{\varepsilon}{h(e)}(1+e\cos\nu)^2, \quad h(e) = \left(1-e^2\right)^{1/2}, \qquad (6.5)$$

where Δ, Λ are coefficients in the right-hand sides of two last equations of (6.3). It is seen from the system (6.4), that δ and λ are slow variables, and ν is semi-slow one.

Applying the modified method of averaging [17], we get:

$$\frac{d\delta}{d\tau} = 0,$$

$$\frac{d\lambda}{d\tau} = \frac{\cos\delta}{2\left(1-e^2\right)^{1/2}}\left(1-\frac{3}{2}\sin^2\theta\right)\left\{\frac{3(A_1^*-A_3^*)}{G^*\left(1-e^2\right)}-\Gamma\right\}.$$

It is seen that the angle of deflection δ of the angular momentum vector **G** from the vertical remains constant in the mentioned approximation, as in the case of asymmetrical satellite.

Taking into account (6.4), we find analytically the law of change of angle λ depending on time τ:

$$\lambda = \lambda_0 + \eta\tau - \frac{3\alpha}{2\beta}\ln\left|\frac{1+\gamma\exp(\beta\tau)}{1+\gamma}\right|,$$

$$\eta = \frac{\cos\delta}{2\left(1-e^2\right)^{1/2}}\left\{\frac{3(A_1^*-A_3^*)}{G^*\left(1-e^2\right)}-\Gamma\right\}, \quad \beta = 2\Gamma_1\left(A_1^*-A_3^*\right),$$

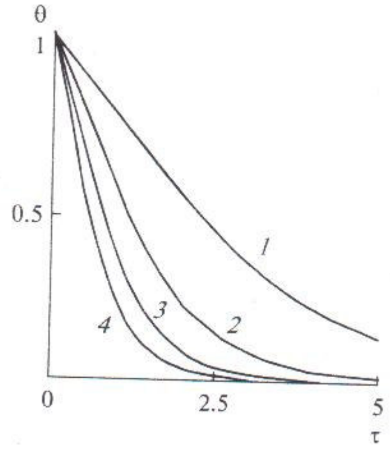

Fig. 9.

$$\gamma = \mathrm{tg}^2\theta_0.$$

The plot of function $\lambda = \lambda(\tau)$ has the form presented in Fig. 8 for the initial value of the nutation angle $\theta(0) = \pi/3$ рад and at the initial value of angle $\lambda = \pi/4$ рад. The curves are constructed for various values of parameter $\beta = -2, -1, 1, 2$. It is seen in the figure that for negative values of parameter β at small times the function $\lambda = \lambda(\tau)$ first increases and then decreases. For positive values of parameter β function $\lambda = \lambda(\tau)$ is descending. At times $\tau > 2.5$ the plots of all functions are almost linear.

In our problem the character of decrease of λ coincides with that obtained in [14, 15] in studying the motion of a satellite with viscous fluid in a cavity under the action of gravitational or light force moments. The angle of orientation of the angular momentum vector **G** in the case considered by us decreases more rapidly.

For the values of parameter $\beta = -0.5, -1, -1.5, -2$ the plots of variation of nutation angle $\theta = \theta(\tau)$ are constructed (Fig. 9). It is seen that the smaller parameter β, the more rapidly the angle $\theta \to 0$, that is, the more "elongated" is the body along the A_3 axis, the more rapidly the satellite tends to the position of stable rotation around this axis.

The character of change of nutation angle θ in the case under consideration is close to that studied in the case of rotation of a satellite with viscous fluid under the action of the moment of light pressure forces [15].

Thus, in the motion of a dynamically symmetric satellite with a cavity, filled with viscous fluid, under the action of the moment of light pressure forces the angular momentum vector **G** remains to be a constant quantity directed at constant angle δ to the orbital plane's vertical. The direction of motion of a tip of vector **G** depends on the shape of a satellite. In case of the satellite "flattened" along the axis of inertia A_3, the tip of vector **G** moves over the sphere of radius G_0 counterclockwise. In this case the nutation angle tends to the limiting value of $\pi/2$rad. When the satellite is dynamically "elongated" along the same axis, the tip of vector **G** moves over the sphere of radius G_0, first clockwise and then counterclockwise, and the nutation angle tends to zero.

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REFERENCES

1. Chernous'ko, F.L., Motion of a Solid Body with Cavities Filled with Viscous Liquid at Small Reynolds Numbers, *Zh. Vychisl. Mat. Mat. Fiz.*, 1965, vol. 5, no. 6, pp. 1049–1070.
2. Beletskii, V.V., *Dvizhenie iskusstvennogo sputnika otnositel'no tsentra mass* (Motion of an Artificial Satellite with Respect to the Center of Mass), Moscow: Nauka, 1965.
3. Chernous'ko, F.L., Attitude Motion of a Satellite under the Action of Gravitational Moments, *Priklad. Mat. Mekh.*, 1963, vol. 27, no. 3, pp. 474–483.
4. Beletskii, V.V., *Dvizhenie sputnika otnositel'no tsentra mass v gravitatsionnom pole* (Attitude Motion of a Satellite in the Gravitational Field), Moscow: Mosk. Gos. Univ., 1975.
5. Karymov, A.A., Stability of Rotational Motion of a Geometrically Symmetric Artificial Satellite of the Sun in the Field of Light Pressure Forces, *Prikl. Mat. Mekh.*, 1964, vol. 28, no. 5, pp. 923–930.
6. Polyakhova, E.N., *Kosmicheskii polet s solnechnym parusom: problemy i perspektivy* (Space Flight with a Solar Sail: Problems and Prospects), Moscow: Nauka, 1986.
7. Sazonov, V.V., Motion of an Asteroid Relative to Its Center of Mass under the Action of the Moment of Light Pressure Forces, *Astron. Vestnik*, 1994, vol. 28, no. 2, p. 95–107.
8. Leshchenko, D.D. and Rachinskaya, A.L., Attitude Motion of a Satellite under the Action of the Moment of Light Pressure Forces in Resisting Medium, *Vestn. Odes. Nats. Univ.*, 2007, vol. 12, no. 7, Mat. Mekh., pp. 85–98.
9. Akulenko, L.D., Leshchenko, D.D., and Rachinskaya, A.L., Evolution of Rotation of a Satellite under the Action of Gravitational Moment in Resistant Medium, *Izv. Ross. Akad. Nauk, Mekh. Tverdogo Tela*, 2008, no. 2, pp. 13–26.
10. Osipov, V.Z. and Sulikashvili, R.S., On Vibrations of a Solid Body with Spherical Cavity Completely Filled with Viscous Liquid in Elliptical Orbit, *Tr. Tbilis. Mat. Inst. Akad. Nauk Gruz. SSR*, 1978, vol. 58, pp. 175–186.
11. Smirnova, E.P., Stabilization of Free Rotation of an Asymmetric Top with Cavities Completely Filled with

Liquid, *Prikl. Mat. Mekh.*, 1974, vol. 33, no. 6, pp. 980–985.

12. Sidorenko, V.V., Evolution of Rotational Motion of a Planet with Liquid Core, *Astron. Vestn.*, 1993, vol.27, no. 2, pp. 119–127.
13. Vil'ke, V.G. and Shatina, A.V., Evolution of Rotation of a Satellite with Cavity Filled with Viscous Liquid, *Kosm. Issled.*, 1993, vol. 31, no. 6, pp. 22–30. [*Cosmic Research*, p. 609].
14. Akulenko, L.D., Leshchenko, D.D., and Rachinskaya, A.L., Evolution of Rotations of a Satellite with Cavity Filled with Viscous Liquid, *Mekh. Tverd. Tela*, 2007, no. 37, pp. 126–139.
15. Akulenko, L.D., Leshchenko, D.D., and Rachinskaya, A.L., Rotations of a Satellite with Cavity Filled with Viscous Liquid under the Action of a Moment of Light Pressure Forces, *Mekh. Tverd. Tela*, 2008, no. 38, pp. 95–110.
16. Volosov, V.M. and Morgunov, B.I., *Metod osredneniya v teorii nelineinykh kolebatel'nykh sistem* (Method of Averaging in Theory of Nonlinear Vibrating Systems), Moscow: Mosk. Gos. Univ., 1971.
17. Akulenko, L.D., Schemes of Higher-Degree Averaging for Systems with Fast and Slow Phases, *Prikl. Mat. Mekh.*, 2002, vol. 66, no. 2, pp. 165–176.
18. Gradstein, I.S. and Ryzhik, I.M., *Tablitsy integralov, summ, ryadov i proizvedenii* (Tables of Integrals, Sums, Series, and Products), Moscow: Nauka, 1971.
19. Landau, L.D. and Lifshits, E.M., *Mekhanika* (Mechanics), Moscow: Nauka, 1973.