Evoluton of rotation of a dynamically symmetric satellite under the action of gravitational and light pressure torques

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EVOLUTION OF ROTATION OF A DYNAMICALLY SYMMETRIC SATELLITE UNDER THE ACTION OF GRAVIITATIONAL AND LIGHT PRESSURE TORQUES

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Abstract

The averaging method is applied to investigate the motion of a satellite with respect to the center of mass caused by gravitation and light pressure. It is assumed that the spacecraft is dynamically symmetric and that its surface is a surface of revolution.

1. Introduction

The attitude evolution of spacecraft under various torques has been studied extensively over the pass few decades. A vast literature (e.g., see [1-13] and the bibliography therein) deals with the investigation of the motion of a satellite about the center of mass under the action of torques of various nature (gravitational, magnetic, light pressure, and the like). Using the averaging method we investigate the motion of a dynamically symmetric spacecraft due to gravitational and light pressure torques.

2. Problem Formulation and Solution

Consider the motion of satellite with respect to its center of mass under the light pressure torques. The center of mass of the spacecraft moves along an elliptic orbit around the Sun. Let us introduce three right-handed Cartesian reference frames centered at the center of mass of the satellite [1, 2]. The reference frame OXYZ moves translationally so that the Y-axis is normal to the plane of the orbit, the Z-axis is co-directed with the position vector of the perihelion of the orbit, and the X-axis is co-directed with the velocity of the satellite center of mass at the perihelion. To define the direction of angular momentum **L** of the satellite about the center of mass in the frame OXYZ, we introduce angles ρ and σ, as it done in [1 – 3]. To construct the reference frame OL_1L_2L associated with the vector **L**, we draw the axis L_1 in the plane OYL so that L_1 is perpendicular to **L** and forms an obtuse angle with the Y-axis. The axis L_2 complements the axes L_1 and L to right-handed trihedral. The axes of the coordinate frame OXYZ rigidly attached to the satellite coincide with the principal central axes of inertia of the satellite. The relative position of the principal central axes of inertia of the satellite and the axes L, L_1 and L_2 are defined by the Eulerian angles φ, ψ, and θ [1-3]. The direction cosines α_{ij} of the x-, y-, and z-axes in the OL_1L_2L reference frame are related to the Eulerian angles φ, ψ, and θ by well-known formulas [1].

We neglect the moment of all forces apart from those of gravitational and of the light pressure. Comparative estimate of gravitational torques for the satellite of the Sun cited in [1]. For the general situation it is shown that the light pressure torques much more then gravitational. In our problem we assume light pressure torque is of order of smallness ε and gravitational torque is of order of smallness ε^2.

Gravitational torque acting on the satellite from side of the Sun has a form [1, 3]

$$M_g = \frac{3\mu}{R^3}\left((C-A)\gamma'\gamma'', (A-C)\gamma\gamma'', 0\right). \tag{1}$$

Here μ is the gravitational parameter of the Sun, R=|**R**|, γ, γ′, γ″ are the cosines between radius-vector **R** and axes x, y, z.

We assume that the surface of the satellite is a surface of revolution with unit vector **k** of the symmetry axis pointing along the z-axis. In this case [1, 4, 5], the torque M_e due to the light pressure forces applied to the satellite is expressed by

$$\vec{M}_c = \left(a_c(\varepsilon_c) R_0^2 / R^2\right) \vec{e}_r \times \vec{k}$$

$$a_c(\varepsilon_s) \frac{R_0^2}{R^2} = p_s S(\varepsilon_s) Z_0'(\varepsilon_s) \tag{2}$$

$$p_c = \frac{E_0}{c} \left(\frac{R_0}{R}\right)^2$$

where $\mathbf{e}_r$ is the unit vector of the position of the satellite center of mass; ε_s is the angle between the vectors $\mathbf{e}_r$ and $\mathbf{k}$ defined so that $|\mathbf{e}_r \times \mathbf{k}| = \sin \varepsilon_s$; R is the current distance between the center of Sun and the center of mass of satellite; R_0 is a fixed value of the variable R, for instance, at the initial time instant; $a_c(\varepsilon_s)$ is the coefficient of the torque due to the light pressure; S is the area of the shadow on the plane normal to the light flux; Z'_0 is the distance between the center of mass of the satellite and the center of pressure; p_c is the light pressure at the distance R from the center of the Sun; c is the velocity of light; E_0 is the magnitude of the radiant energy flux at the distance R_0 from the center of the Sun.

In what follows, we assume that $a_c = a_c(\cos \varepsilon_s)$ [1] and approximate the function a_c by polynomials in $\cos \varepsilon_s$. The light pressure torque has a force function depending only on the orientation of the symmetry axis of the body [1]. We represent the function $a_c(\cos \varepsilon_s)$ in the form

$$a_c = a_0 + a_1 \cos \varepsilon_s . \tag{3}$$

Now we consider only the second term of the expansion. Since the gravitational and light pressure torque have a force function, the equations of the perturbed motion of the satellite can be represented in the variables L, ρ, σ, φ, ψ, and θ in the form [3]

$$\begin{aligned}
\dot{\sigma} &= (L \sin \rho)^{-1} \frac{\partial U}{\partial \rho} \\
\dot{\rho} &= -(L \sin \rho)^{-1} \frac{\partial U}{\partial \sigma} + L^{-1} ctg\rho \frac{\partial U}{\partial \psi} \\
\dot{L} &= \frac{\partial U}{\partial \psi} \\
\dot{\theta} &= -(L \sin \theta)^{-1} \frac{\partial U}{\partial \varphi} + L^{-1} ctg\theta \frac{\partial U}{\partial \psi} \\
\dot{\varphi} &= L \cos \theta \left(C^{-1} - A^{-1}\right) + (L \sin \theta)^{-1} \frac{\partial U}{\partial \theta} \\
\dot{\psi} &= LA^{-1} - L^{-1} \left(\frac{\partial U}{\partial \rho} ctg\rho + \frac{\partial U}{\partial \theta} ctg\theta \right)
\end{aligned} \tag{4}$$

The force function U depends on time (via the true anomaly ν(t)) and the direction cosines α_3, β_3, and γ_3 of the z-axis in the reference frame OXYZ; it has the form $U = U(\nu(t)\ \alpha_3, \beta_3, \gamma_3)$. System (4) must be completed by the equation describing the variation of the true anomaly in the time. In the case of circular orbit (e=0) we have

$$\frac{d\nu}{dt} = \omega_0, \omega_0 = \frac{2\pi}{T_0} = \left[\mu P^{-3}\right]^{1/2} . \tag{5}$$

Here ω_0 is the average angular velocity of motion of the center of mass along the circular orbit; T_0 is the period of revolution of the satellite; e and P are the eccentricity and the focal parameter of the orbit, respectively, μ is the product of the gravitational constant by the mass of the Sun.

We consider the force function consists of two terms caused by the influence of gravitational torque and light pressure torque $U=U_g+U_c$. The force function caused by the influence of a gravitational torque has a form [3]

$$
\begin{aligned}
U_g &= \frac{3\omega_0}{2}(A-C)\gamma''^2 \\
\gamma'' &= \alpha_3 \sin\nu + \gamma_3 \cos\nu
\end{aligned} \tag{6}
$$

The force function caused by the influence of a light pressure torque has a form

$$
U_c(\cos\varepsilon_c) = -\frac{R_0^2\omega_0^{4/3}\alpha_1}{2\mu^{2/3}}(\gamma_3\cos\nu + \alpha_3\sin\nu)^2. \tag{7}
$$

The direction cosines α_3 and γ_3 can be expressed via ρ, σ, θ, and ψ according to well-known formulas [1].

We investigate the motion of a dynamically symmetric satellite in assumption that angular velocity ω of the motion of the satellite about the center of mass much more than angular velocity of orbital motion ω_0, i.e. $\varepsilon=\omega_0/\omega\sim A\omega_0/G\ll 1$. Assume that the coefficient of the light pressure is of the order of ε and gravitational torque is of the order of ε^2, where ε is a small parameter, $0<\varepsilon\ll 1$.

We investigate the solution of the system (4)-(7) for small ε on a large time interval $t\sim\varepsilon^{-2}$. We perform the averaging with respect to ψ and ν [1, 3].

We turn to dimensionless variables and denote $t'=\Omega_0 t$, $L'=L/L_0$. We introduce small parameters $\omega_0^2(A-C)/(L_0\Omega_0)=\varepsilon^2$, $R_0^2\omega_0^{4/3}\alpha_1\Big/\left(\mu^{2/3}L_0\Omega_0\right)=\varepsilon$ where Ω_0 is angular velocity of the motion of satellite relative to its center of mass in the initial moment of time, L_0 is a value of the angular momentum of the satellite in the initial moment of time.

After the averaging by ψ we have

$$
\begin{aligned}
L' &= const \\
\frac{d\rho}{dt'} &= \frac{1}{L'}\left(1-\frac{3}{2}\sin^2\theta\right)\sin\rho\cos(\sigma-\nu)\sin(\sigma-\nu)\left(3\varepsilon^2-\varepsilon\right) \\
\frac{d\sigma}{dt'} &= \frac{1}{L'}\left(1-\frac{3}{2}\sin^2\theta\right)\cos\rho\cos^2(\sigma-\nu)\left(3\varepsilon^2-\varepsilon\right)
\end{aligned} \tag{8}
$$

System (8) is a system of the form

$$
\xi = \varepsilon\Sigma_1(\xi) + \varepsilon^2\Sigma_2(\xi). \tag{9}
$$

We use method of successive approximations by the power of a small parameter ε [14] for investigation system (8).

We solve at first the equations of the first approximation

$$
\begin{aligned}
\dot{\xi}_1 &= \varepsilon\Sigma_1(\xi_1) \\
\frac{d\xi_1}{d\tau} &= \Sigma_1(\xi_1)
\end{aligned} \tag{10}
$$

Here $\tau=\varepsilon t$.

After the averaging by ν system (8) takes a form

$$\frac{d\rho}{dt'} = 0$$
$$\frac{d\sigma}{dt'} = -\frac{\varepsilon}{2L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\cos\rho \tag{11}$$

From where we receive

$$\rho = \rho_0$$
$$\sigma = -\frac{\varepsilon}{2L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\cos\rho_0 t + \sigma_0 \tag{12}$$

Here ρ_0, σ_0 are initial values of the angles ρ and σ.

After that we integrate system (10) or (11) we look for the solution of the system of the second approximation for $\xi \approx \xi_2$

$$\frac{d\xi_2}{d\tau} = \Sigma_1(\xi_2) + \varepsilon\Sigma_2(\xi_2) \tag{13}$$

in the form

$$\xi_2(\tau) = \xi_1(\tau) + \varepsilon\delta\xi_1(\tau) + \varepsilon^2 \ldots \tag{14}$$

In our problem we must solve the equations of the form

$$\frac{d(\delta\rho_1)}{d\tau'} = -\frac{1}{2L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\cos\rho_0 \times$$
$$\times \sin\left[-\frac{\tau'}{2L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\cos\rho_0 + 2\sigma_0 - 2\nu\right]\delta\rho_1 + \tag{15}$$
$$+\frac{3}{2L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\sin\rho_0 \sin\left[-\frac{\tau'}{L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\cos\rho_0 + 2\sigma_0 - 2\nu\right]$$

$$\frac{d(\delta\sigma_1)}{d\tau'} = \frac{1}{L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\cos\rho_0 \times$$
$$\times \sin\left[-\frac{\tau'}{L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\cos\rho_0 + 2\sigma_0 - 2\nu\right]\delta\sigma_1 + \tag{16}$$
$$+\frac{3}{L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\cos\rho_0 \cos^2\left[-\frac{\tau'}{2L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\cos\rho_0 + \sigma_0 - \nu\right]$$

Here $\tau' = \varepsilon t'$.

We solve the linear nonhomogeneous differential equations of the first order (15), (16) and we define

$$\delta\rho_1 = C_1 \exp\left\{-\frac{1}{2}\cos\left[-\frac{\tau'}{L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\cos\rho_0 + 2\sigma_0 - 2\nu\right]\right\} + 3tg\rho_0 \tag{17}$$

$$\delta\sigma_1 = 3C_2 + \exp\left\{\frac{1}{2}(1+3\varepsilon)\sin\left[\frac{1}{8L'}\cos\rho_0(-1+3\varepsilon)\tau'(2+3\cos 2\theta) + \right.\right.$$
$$\left.\left. + 2\sigma_0 - 2\nu\right] - \frac{\cos\rho_0(2+3\cos 2\theta)\tau'}{16L'} + (\sigma_0 - \nu)(1+3\varepsilon)\right\} \tag{18}$$

Thus we find the solutions of the system of the second approximation in the form

$$\rho = \rho_0 + \varepsilon C_1 \exp\left\{-\frac{1}{2}\cos\left[-\frac{\tau'}{L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\cos\rho_0 + 2\sigma_0 - 2\nu\right]\right\} + 3\varepsilon tg\rho_0 \tag{19}$$

$$\sigma = -\frac{\varepsilon}{2L'}\left(1 - \frac{3}{2}\sin^2\theta\right)\cos\rho_0 t + \sigma_0 + 3\varepsilon C_2 +$$
$$+ \varepsilon\exp\left\{\frac{1}{2}(1+3\varepsilon)\sin\left[\frac{1}{8L'}\cos\rho_0(-1+3\varepsilon)\tau'(2+3\cos 2\theta) + \right.\right. \tag{20}$$
$$\left.\left. + 2\sigma_0 - 2\nu\right] - \frac{\cos\rho_0(2+3\cos 2\theta)\tau'}{16L'} + (\sigma_0 - \nu)(1+3\varepsilon)\right\}$$

3. Conclusions

We have investigated the evolution of the rotation of a dynamically symmetric satellite acted upon by the gravitational torque and light pressure torque. New properties of the rotations of the satellite are established. This case is of interest both theoretically and in applications.

References

[1] V.V. Beletskii. *Motion of an Artificial Earth Satellite About its Center of Mass.* Nauka, Moscow, 1965 (in Russian).

[2] F.L. Chernousko. On the motion of a satellite about its center of mass under the action of gravitational torques, *PMM [Applied Mathematics and Mechanics]*, 27 (3), 474 – 483, 1963.

[3] V.V. Beletskii. *Motion of a Satellite About its Center of Mass in the Gravitational Field*, MSU, Moscow, 1975 (in Russian).

[4] A.A. Karymov, Stability of rotations of a geometrically symmetric artificial satellite of the Sun in the field of light pressure forces. *PMM USSR [Applied Mathematics and Mechanics]*, 28 (5), 923 – 930, 1964.

[5] V.K. Abalakin, E.P. Aksenov, E.A. Grebenikov etc, *A Handbook on Celestial Mechanics and Astrodynamics*. Nauka, Moscow, 1976 (in Russian).

[6] E.N. Polyakhova, *Cosmic Flight with Solar Sale: Problems and Perspective*. Nauka, Moscow, 1986 (in Russian).

[7] V.V. Sidorenko, About the rotational motion of a spacecraft with solar stabilizer. *Cosmic Investigations*, 30 (6), 780 – 790, 1992.

[8] V.V. Sazonov, Motion of asteroid relative to the center of mass under the action of light pressure torque. *Astronomical Vestnik*, 28 (2), 95 – 107, 1994.

[9] D.D. Leshchenko, A.S. Shamaev, Motion of a satellite relative to the center of mass under the action of moments of light pressure forces, *Izvestiya AN SSSR. Mekhanika Tverdogo Tela (Mechanics of Solids)*, 1, 14–21, 1985.

[10] L.D. Akulenko and D.D. Leshchenko, Evolution of rotation of a nearly dynamically spherical triaxial satellite under the action of the light pressure torques, *Izvestiya RAN. Mekhanika Tverdogo Tela (Mechanics of Solids)*, 2, 3 – 12, 1996.

[11] D.D. Leshchenko, Evolution of rotation of a triaxial body under the action of the light pressure torques. *Izvestiya RAN. Mekhanika Tverdogo Tela (Mechanics of Solids)*, 6, 17 – 26, 1997.

[12] L.D. Akulenko, D.D. Leshchenko, S.G. Suksova etc, The motion of a satellite about its center of mass under the action of gravitational and light pressure torques. *Mekhanika Tverdogo Tela (Donetsk)*, 34, 95 – 105, 2004.

[13] L.D. Akulenko, D.D. Leshchenko, S.G. Suksova etc, Evolution of rotation of a nearly dynamically spherical triaxial satellite under the action of gravitational and light pressure torques, *Izvestiya RAN. Mekhanika Tverdogo Tela (Mechanics of Solids)*, 4, 97 – 107, 2006.

[14] V.M. Volosov and B.I. Morgunov, *Averaging Method in the Theory of Nonlinear Oscillatory Systems*. MGU, Moscow, 1971 (in Russian).