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DEFORMATION-STRENGTH MODEL APPLICATION AT THE DETERMINING OF STRESS-STRAIN STATE OF REINFORCED CONCRETE STRUCTURES

В.М. Карпюк, А.І. Костюк, Ю.А. Сьоміна, Д.С. Даниленко. **Застосування деформаційно-силової моделі при визначенні напружено-деформованого стану залізобетонних конструкцій.** Робота розглядає можливість та доцільність застосування деформаційно-силової моделі при вивченні напружено-деформованого стану залізобетонних конструкцій. Спираючись на основні положення механіки твердого деформівного тіла та реальні стадії роботи залізобетонних елементів, авторами узагальнено та встановлено, що реальний стан залізобетонної конструкції не може бути відображений тільки епюрою напружень або тільки епюрою деформацій. Метою статті є розгляд роботи бетонних і залізобетонних елементів в ключі як силової, так і деформаційної моделі. Теорія опору залізобетонних конструкцій, як і раніше, залишається направленою на якнайточніше визначення чотирьох найважливіших задач: точний розрахунок навантаження, при якому з'являються перші тріщини; визначення ширини розкриття тріщин в експлуатаційній стадії, починаючи з моменту їх появи; розрахунок жорсткості та величини прогинів, в тому числі гранично допустимих; визначення максимально можливої несучої здатності (міцності або стійкості). Реальний стан залізобетонної конструкції не може бути відображений тільки епюрою напружень або тільки епюрою деформацій. Це може бути зроблено лише при спільному використанні обох епюр. При цьому, узагальнена модель деформування елемента повинна бути здатною в однаковій мірі відображати як характер зростання відносних деформацій матеріалів, так і процес постійного перерозподілу напружень в них, особливо на стадіях, близьких до граничної рівноваги.
Ключові слова: залізобетонний елемент, напружено-деформований стан, деформаційно-силова модель, епюра

V.M. Karpiuk, A.I. Kostiuk, Ya.A. Somina, D.S. Danilenko. **Deformation-strength model application at the determining of stress-strain state of reinforced concrete structures.** The work considers the possibility and expediency of deformation-strength model application in the study of stress-strain state of reinforced concrete structures. Relying on the basic provisions of the mechanics of solid deformable body and the real stages of work of reinforced concrete elements, it is generalized and established by the authors that the real state of reinforced concrete structures cannot only be described by stresses diagrams or only by strain diagram. Aim of the paper is consideration of concrete and reinforced concrete elements' work as a force and deformation model. The theory of reinforced concrete structures resistance, as before, is aimed at a precise definition of the four major problems: accurate calculation of the load at which the first cracks appear; determination of the width of the cracks in the operational phase, starting from the moment of their appearance; calculation of rigidity and deflections, including the maximum permissible; definition of maximum possible bearing capacity (strength or stability). The real state of the reinforced concrete structure cannot be displayed only by stress distribution or deformations diagrams. This can only be done when used in conjunction with both diagrams. In this case, the generalized model of the element deformation should be able to equally reflect both the nature of the growth of relative deformation of materials, and a process of continuous redistribution of stresses in them, especially at the stages that are close to the limit equilibrium.
Keywords: reinforced concrete element, stress-strain state, deformation-strength model, diagram

Introduction. Theory of concrete and reinforced concrete continues to develop in the direction of compliance with generally accepted principles and preconditions of solids mechanics. Even though the problem of the general theory of resistance of concrete and reinforced concrete creation is far from perfect, there are already developed certain ways to solve it.

In terms of strain-force model of reinforced concrete structures, resistance force and other effects mean a prototype of a real process of deformation of concrete and reinforced concrete elements and structures reproduced by some of the generalized state diagram. It should be emphasized that the basis of modern deformation models of reinforced concrete elements and structures resistance to power action laid discretization design schemes and their representation in the form of a set of elements of certain structural levels.

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The aim of the paper is consideration of work of concrete and reinforced concrete elements as a force and deformation model.

Materials and Methods. The theory of resistance of reinforced concrete structures, as before [1], is aimed at a precise definition of the four major problems:

– accurate calculation of the load at which the first cracks appear;

– determination of the width of the cracks in the operational phase, starting from the moment of their appearance;

– calculation of rigidity and deflections, including the maximum permissible;

– definition of maximum possible bearing capacity (strength or stability).

Thus, under the influence of external loads or impacts, the internal forces in the most intense section of reinforced concrete elements, as well as deformation of concrete in its extreme fiber, increases from zero to some limits. With double-digit stress diagram, the section usually goes through three typical stages of stress-strain state.

Stage 1 (operation without fractures) is observed at relative elongation of the area of concrete tension, less than the limit value of ε_{ctu}. At this stage the calculated deflection value and a rigidity of low reinforced structures operating without cracks in the tension zone are calculated.

Stage 1*a* occurs at the moment when the relative elongation deformation of concrete extreme fibers reaches the limit values of ε_{ctu}. Concrete starts to burst on and off from work, causing a reinforcement to work hard. An accurate assessment of stress-strain state of the cross sections at this stage will allow determining the force at which the first cracks appear in the concrete tension area and calculating the stiffness of yet solid (without cracks) section.

Stage 2 occurs after the appearance of the first cracks in the concrete tension area. It is considered as the main working stage of deformation of reinforced concrete bent elements. At this stage researches can determine the width of the crack opening and calculate the stiffness and the magnitude of the deflection of reinforced concrete elements under the action of operating loads.

Stage 2*a* represents the limit state. It occurs when the stress in the tension reinforcement reaches the representative strength values on the verge of its yield f_{yk} or extreme fiber stress of compressed concrete – representative values of the compressive strength of f_{ck}, i.e., in the cross section reinforced concrete element begins to form so-called "plastic hinge" (starts the destruction).

The final stage 3 represents the limit stress-strain state of reinforced concrete elements and characterizes its complete destruction. It occurs when the balance of forces in the most stress section element can no longer be ensured. At the same time, the relative deformation of the extreme fibers, compressed with concrete, reaches the value of $\varepsilon_{cu} > \varepsilon_{c1}$, and tension reinforcement can work both before and on the border of yield: $\varepsilon_s < \varepsilon_{s0}$, $\varepsilon_{s0} \leq \varepsilon_s \leq \varepsilon_{suk}$.

Constructions researches on stages 2*a* and 3 make it possible to determine the real value of the destroying efforts that should be displayed in the standards.

As we know from mechanics of solid deformable body for the task we need two main and four additional conditions, there are two equilibrium equations for planar systems in a normal section, as well as auxiliary terms in the form of strain distribution law along the section height of the element $1/r=(\varepsilon_c+\varepsilon_s)/d$, physical dependence between stresses in the reinforcement $\sigma_s=f(\varepsilon_c)$, and the compressed and tensioned zones of the concrete $\sigma_c=f(\varepsilon_c)$ and $\sigma_{ct}=f(\varepsilon_{ct})$.

It should be noted that the mapping of stages of the stress-strain state of reinforced concrete structures for force model of the previous standards [2] has a number of serious shortcomings:

– simplified method of accounting for concrete plastic deformations in the form of a rectangular stress distribution;

– adopted in [2] approach of calculating strength of reinforced concrete elements cannot be directly implemented in the calculations of their hardness and fracture toughness;

– accounting method of the impact of the work of tensioned concrete between and above the cracks on the overall stress-strain state of reinforced concrete element weakly reflects the redistribution of forces between the concrete in tension and tensioned reinforcement;

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– a characteristic of strength tensile reinforcement on the verge of its yield f_{yk} or the representative value of concrete compressive strength of f_{ck} in its extreme fibers cannot act as the exhaustion criteria of bearing capacity of reinforced concrete structures.

Deformation model, as opposed to force one, more accurately reflects the stress-strain state of reinforced concrete elements in a limit stage. However, in these models has not yet been developed a single general criterion of the exhaustion of bearing capacity.

If we assume that the real model of deformation of concrete and reinforced concrete structures is [3] the deformation force in nature, in its framework, the only common criterion of exhaustion of bearing capacity can act as the moment of imbalance of power that is secured by an extreme criterion of bearing capacity $dM/d(1/r)$.

Loss of the bearing capacity of normal cross sections by reinforced concrete elements is characterized by a violation of one of the two known equilibrium equations $\Sigma N=0$ and $\Sigma M=0$. More rigid is the second equation, which implies defining the condition of limit equilibrium

$$M_{Ed} \le M_u, \qquad (1)$$

where M_{Ed} – estimated value of the bending moment on the external load;

M_u – limit value of moment of internal forces in the cross section of reinforced concrete element.

In the design of concrete and reinforced concrete structures, the maximum bearing capacity M_u or N_u by $dM/d(1/r)=0$ or $dN/d\varepsilon=0$ is calculated according to [3].

The characteristic that links the strength (M, N) and deformational ($1/r$, ε) parameters, can be stiffness of element in a certain cross section. The knowledge patterns of change in the rigidity are critical not only in the calculation of reinforced concrete structures for deflections and crack resistance, but also in determining of its load-bearing capacity.

It is known that the rigidity of the concrete or reinforced concrete element is an integral characteristic. Obviously, the axial compression or tension of concrete element or its compression-tension with small or occasional eccentricities varies primarily or only by changing the concrete strain module (fig. 1, *a*), since all the geometric parameters of such elements remain unchanged:

$$D_{cc} = E_{cc}^{\text{int}} I_{cc}, \quad D_{ct} = E_{ct}^{\text{int}} I_{ct}, \qquad (2)$$

where $E_{cc(t)}^{\text{int}}$ − integrated (averaged) module of compressed or stretched concrete strain;

$I_{cc(t)}$ − moment of inertia of the compressed or tensioned concrete section.

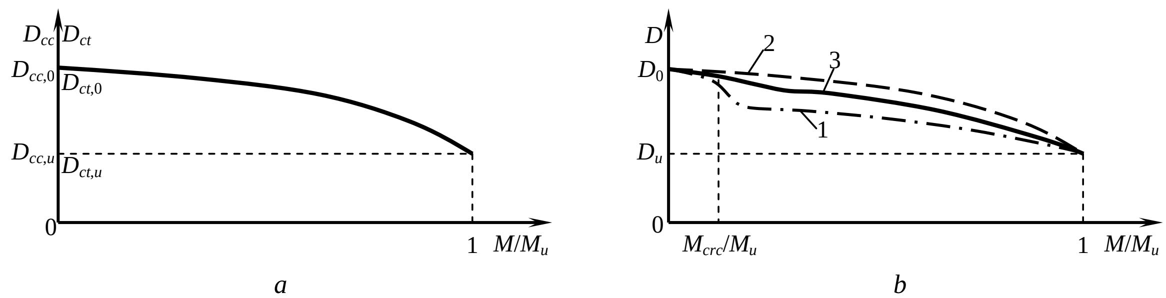

Fig. 1. Modifying stiffness of the compressed (tensioned) concrete element with a small eccentricity (a) and the diagram of the integral (averaged) bending stiffness of a reinforced concrete element (b) in section:1 – with crack; 2 – between cracks; 3 – averaged in block between cracks

With double-digit stress diagram in the reinforced concrete elements there can appear and develop cracks, traction of tension reinforcement with concrete breaks and its gradual exclusion from work takes place. The value of the integral rigidity of these elements consists of stiffness of compressed and stretched zones of concrete, compressed and stretched reinforcement:

$$D = E_{cc}^{\text{int}} I_{cc} + E_{cc}^{\text{int}} I_{ct} + \sum E_{sc} I_{sc} + \sum E_{st} I_{st} \, . \qquad (3)$$

The function of rigidity in a section with a crack (fig. 1, *b*) should reflect the relatively rapid (almost sudden) exclusion from work of the concrete in tension and the corresponding redistribution of stresses in the tension zone from the concrete to reinforcement.

In the block between the cracks a reinforced concrete element stiffness varies not by geometric but by deformation characteristics (modulus of concrete strain) but may also be reflected by the expression (3).

As it is known, when bearing capacity is exhausted, the element section is not destroyed, but concrete in certain volume or reinforcement in a certain area is. Thus, the stress-strain state of reinforced concrete elements at this site should describe the characteristics of a calculated (averaged) section of the block between the cracks, including the averaged integral rigidity, which will change as the expression (3), relatively smoothly, with no apparent jumps and ruptures of function.

Calculation of the averaged integral stiffening of the reinforced concrete structure by the expression (3) is a difficult task with many unknowns associated not only with the deformation characteristics of the materials, but also with geometric section parameters. Therefore, in practice averaged integral stiffness is suitably calculated in its classical expression:

$$D = M / (1/r). \tag{4}$$

Obviously, connection between stiffness and emerging of internal forces in the section element and its curvature has to be nonlinear, capable of reproducing its stress-strain state only, with simultaneous use of force and deformation characteristics.

In real conditions, even if the axial load concrete and reinforced concrete elements stiffness is changing both due to the deformation properties of materials (E), and due to the geometric parameters of cross-section (I), then $M/(1/r)=EI$.

In general, the stiffening of element is associated not only with the level of efforts exposure M/M_u, but also with its deformation level $(1/r)/(1/r_u)$. Therefore, the force, which must be taken by element, works [3] and serves for calculations, depending on its level of deformation, by the formula:

$$M = \frac{E_{c0} I_{red,0} \cdot 1/r - M_u \left(\frac{1/r}{1/r_u} \right)^2}{1 + \left(\frac{E_{c0} I_{red,0}}{M_u} - \frac{2}{1/r_u} \right) \frac{1}{r}}, \tag{5}$$

where M_u – bearing capacity of concrete or reinforced concrete bar;

$1/r$ − element curvature in limit state;

$E_{c0} I_{red,0}$ − initial reduced stiffness of the concrete or a reinforced concrete element section.

It is obvious that the expression (5) describes the state diagram of an element or a structure (fig. 2) with the ascending and descending deformation branches. In this case, the descending branch of the diagram is not possible for statically determinated reinforced concrete structures and concrete elements because achievement of the ultimate state in the most intense section will lead to geometric variability and destruction.

Structural analysis of the expression (5) shows that it describes a smooth and monotonic function of the ascending and descending branches of the diagram, and therefore can be used unconditionally for reinforced concrete structures, operating without formation of cracks in the tension zone. With the emergence of crack their stiffness will change more rapidly due to the accelerated elimination of the work of concrete in tension and increasing of the curvature $1/r$.

Obviously, the rapid drop in stiffness (increasing of curvature) occurs as a result of intense changes in the geometric characteristics of its cross-section. The overwhelming majority of foreign scientists associated stiffness of reinforced concrete structures with the so-called effective moment of inertia, which depends on the level of loading. Typically, adjusting the effective moment of inertia Ie is carried out using different degree dependencies [4 – 11].

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Likewise, it is advisable to adjust the curvature itself and reinforced concrete structures with cracks in the tension zone using functions like [3]:

$$\psi_i = 1 + m \cdot M / M_u (1 - M / M_u) \qquad (6)$$

where m – option, which is recommended [3] to be taken, depending on character of destruction:

m=3 for the work reinforcement yield in low reinforced elements ($\rho_l \leq 1.5$ %);

m=2 for the yield in normally reinforced elements (1.5 % < ρ_l <3.5 %);

m=1.5 for concrete smashing in over-reinforced elements (ρ_l > 3.5 %) and in structures with high-strength reinforcement.

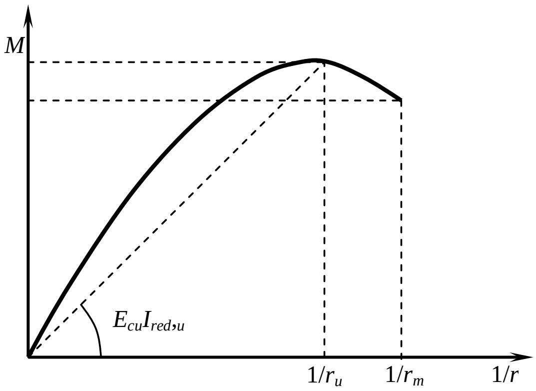

Fig. 2. Relation between the curvature and the moment in the reinforced concrete structure (state diagram)

If in the equation (5) we replace $1/r$ with the adjusted (real) curvature $1/r^*$ of a reinforced concrete element with cracks by$1/r=(1/r^*)\psi_i$, the traditional entry of the phase diagram can be preserved in practical calculations in the form of:

$$M = \frac{E_{c0} I_{red,0} \cdot \dfrac{1/r^*}{\psi_i} - M_u \left(\dfrac{1/r^*}{\psi_i \cdot 1/r_u} \right)^2}{1 + \left(\dfrac{E_{c0} I_{red,0}}{M_u} - \dfrac{2}{1/r_u} \right) \cdot \dfrac{1/r^*}{\psi_i}}. \qquad (7)$$

In the work [3] is shown that seeking a universal function $\sigma_c = f(\varepsilon_c)$ based on the results of experimental studies of standard samples is impossible, because in nature there are no two equivalent elements of reinforced concrete, the stress-strain state of which would be the same. Therefore, we should look for a universal function, not for a specific stress-strain diagram and phase diagram of a concrete or a reinforced concrete element $M-(1/r)$, which can be represented by irregular fractional-rational functions (5) or (7), since they allow us to describe the stress-strain state of a bent and centrally or eccentrically compressed or tensed concrete or a reinforced concrete element. Thus, it is evident that the material deformation diagram is a state diagram of standard sample under the standard conditions of testing. If we apply $M-(1/r)$ limit equilibrium hypothesis and extreme criterion of bearing capacity $dM/d(1/r)=0$ to the generalized state diagram of reinforced concrete element, we can get the actual diagram of deformation of the compressed or tensile concrete.

Results. So, the real state of the reinforced concrete structure cannot be displayed only by stress distribution or deformations diagrams. This can only be done when used in conjunction with both diagrams. In this case the generalized model of the element deformation should be able to equally reflect both the nature of the growth of relative deformation of materials and a process of continuous redistribution of stresses in them, especially at the stages that are close to the limit equilibrium.

Conclusions. Thus, analyzing the given information it is determined by the authors that the real model of concrete and reinforced concrete elements and structures is always a deformation-force model and cannot be purely deformation or purely force.

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