Evolution of Motions of a Rigid Body About its Center of Mass

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Preface

The problem of motion of a rigid body about a fixed point is one of the classical problems of mechanics, the study of which in the eighteenth to nineteenth centuries is connected with the names of Euler, Lagrange, and Kovalevskaya. These scientists discovered three cases in which a complete integration of the motion equations of a rigid body is possible; these cases are named after their discoverers. The solution in Euler's case describes the motion of a free rigid body. Lagrange's case corresponds to a heavy rigid body having a dynamic symmetry, with the center of gravity of the body and the fixed point lying on the dynamic symmetry axis. Kovalevskaya's case occurs when there is a special relation between the inertia moments. Later on, the research in this field was mainly aimed to finding first integrals and particular solutions in the problem of the rigid body dynamics.

The interest to the problems of the rigid body dynamics has significantly increased in the second half of the twentieth century in connection with the development of rocket and space technologies, the increasing speed and maneuverability of aircrafts, and the creation of gyroscopic systems. The study of the motion of satellites and space vehicles about the center of mass is important for creating the systems of orientation control, stabilization of motion, and, ultimately, solving the practical problems of astronautics.

A satellite or a spacecraft in its motion about the center of mass is affected by the moments of forces of various physical nature. It is influenced by the gravitational, aerodynamic, electromagnetic torques, the torques due to the light pressure, as well as the torques due to the motion of some masses inside the body. These motions may have various causes: the presence of fluid in the cavities in the body (e.g., liquid fuel or oxidizer in the tanks of a rocket), structural flexibility (elastic flexibility) of the flying vehicle, the presence in the body of rotating masses (rotors, gyroscopes, gyrodines), the complex internal structure (in the case of natural celestial bodies), and also the movements of the crew members (in the case of a crew vehicle).

The above-mentioned torques, acting on the body, are often relatively small and can be considered as perturbations. Therefore, it is natural to use the asymptotic methods of small parameter or the perturbation techniques to analyze the dynamics of rigid body under the action of the applied torques.

This monograph presents the results of the authors' research on the dynamics of rigid body motion about its center of mass. The authors consider the evolution of these motions under the influence of various perturbation torques. The basic method applied in the studies is the Krylov–Bogolubov asymptotic averaging method.

The main content of the book is preceded by the literature review, where the studies with the topics close to the subject of the monograph are briefly described.

The book consists of 11 chapters.

In Chap. 1, the fundamentals of the rigid body dynamics are briefly presented. The basic concepts are introduced; the fundamental kinematic and dynamic equations are formulated.

Chapter 2 is devoted to the inertial motion of a rigid body, that is, to Euler's case. This motion serves as a reference or generating one for the majority of perturbed motions considered in the book, because, with the action of small perturbation torques, the motion turns out to be close to Euler's motion over short intervals of time. Some information about Euler's motion necessary for further consideration is given.

Lagrange's case is described in Chap. 3. This case is used as a reference one in Chap. 11. The necessary relations for this motion are presented, including its particular cases: regular precession and fast rotation.

Chapter 4 contains the basics of the averaging method, widely used in the book. The notions of a system in a standard form and a system with rapidly rotating phase are introduced; some accuracy estimates for the method are indicated. The application of the averaging method to the equations of the perturbed motion that is close to the motion in Euler's case is discussed. It is the procedure of averaging over the motion in Euler' case that allows studying the evolution of the satellite motions in the case of various perturbations. Next, the equations of the perturbed motion that is close to Lagrange's case are also considered.

Chapter 5 is dedicated to the description of various perturbation torques, acting on a rigid body. We present some relations, necessary for further reasoning, for the gravitational torques acting on a satellite, for the torques of resistance forces of the external medium, for the torques of the light pressure forces, as well as for the torques due to the presence of a viscous fluid in a cavity of the rigid body. Different cases are considered of the influence of internal masses on the body motion: the presence of elastic and dissipative elements and the viscoelastic properties of the moving body.

In the subsequent Chaps. 6–10, we study the perturbed motions of a rigid body under the influence of various external and internal torques. The corresponding expressions for the perturbation torques are taken from Chap. 5. As a reference (generating) motion in Chaps. 6–10, we use the motion in Euler's case, and the averaging over this motion is performed according to the asymptotic procedure explained in Chap. 4.

Chapter 6 is devoted to the satellite motion on an elliptic orbit about the center of mass under the action of gravitational torques. We investigate the cases of a satellite with the close moments of inertia and the fast rotations of a satellite with arbitrary moments of inertia. The planar oscillatory and rotational motions of a satellite about its center of mass on an elliptic orbit are considered separately.

In Chap. 7, we consider the motions of a rigid body with a cavity filled with a viscous fluid. As a result of the conducted asymptotic analysis, a solution is obtained which describes, in a nonlinear setting, the evolution over a significant time interval of the motion of a body having a cavity with a high-viscosity fluid. It is demonstrated that the dynamics of a body with a cavity containing a viscous fluid is equivalent to the dynamics of a gyrostat, carrying the rotors interacting with the body through the linear resistance forces.

Chapter 8 is devoted to the motion of a rigid body in a resistant medium that acts on the body by the torques depending on its angular velocity. We consider also the case of simultaneous influence of the moments of gravitational and resistance forces. The motion of a satellite about its center of mass under the action of gravitational torque and the torque of resistance forces is studied.

In Chap. 9, various cases are studied of the motion of a rigid body having internal degrees of freedom. We consider the motions of a body that contains linear elastic and dissipative elements. In particular, the motions are investigated of a body carrying masses which are attached to it by means of elastic forces with linear or quadratic damping. This situation simulates the presence of loosely fixed components on a spacecraft, having a significant influence on its movement relative to the center of mass.

The influence of the torque of the light pressure forces on the motion of a satellite of the Sun about the center of mass is considered in Chap. 10. We study the evolution of angular motions in a number of cases.

Chapter 11 is dedicated to the perturbed motions of a rigid body that are close to Lagrange's case. As a generating motion, we use the motion described in Chap. 3. By means of averaging over the motion in Lagrange's case, we analyze the perturbed motions under the action of linear dissipative torques.

For all cases of motion considered in the book, we present and analyze the basic equations of motion, perform an averaging procedure, and obtain the averaged equations, which, being significantly simpler than the original ones, describe the motion over a large time interval. We present the accuracy estimates for the asymptotic procedure. As a result of analysis and solution of the obtained equations, we establish some quantitative and qualitative specific features of the motions and provide a description of the evolution of the body motion. The presentation is illustrated by numerous examples.

The authors hope that the book will be of interest for the scientists working in the field of mechanics and applied mathematics, engineers, postgraduate students, and students of the corresponding specialties.

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Survey of Literature

The problem of the motion of a rigid body about a fixed point is one of the classical problems of mechanics. In 1758, L. Euler [1] obtained a solution of this problem for the case of a free rigid body when the center of mass coincides with the fixed point. In the year 1788, J. Lagrange [2] investigated the motion of a heavy rigid body in the case when the ellipsoid of inertia about the fixed point is an ellipsoid of revolution, whereas the center of mass of the rigid body lies on the axis of symmetry of this ellipsoid.

After Lagrange, the research of rotation of a rigid body about a fixed point continued, but it was only in 1889 when S.V. Kovalevskaya discovered one more case [3], in which the solution can be obtained for arbitrary initial conditions. Besides, several cases were discovered, for which some particular solutions of the motion equations were obtained. Among them are the cases of W. Hess, D.K. Bobylev, V.A. Steklov, D.N. Goryachev, S.A. Chaplygin, G. Grioli, and others.

In the second half of the twentieth century, new forms of equations of a rigid body were obtained; the methods of their investigation were developed. As a result, new solutions for the problem of the rigid body motion about a fixed point were constructed. The main results in this area were obtained by P.V. Kharlamov, E.I. Kharlamova, and other researchers, first of all, the representatives of the Donetsk school of mechanics.

A review of the obtained results, their classification, and detailed bibliography can be found in the works by P.V. Kharlamov [4]; I.N. Gashenenko, G.V. Gorr, and A.M. Kovalev [5]; and A.V. Borisov and I.S. Mamaev [6].

Some general issues of the rigid body dynamics are considered in the monographs [7, 8].

Practical problems require studying more complex motions of a rigid body. A number of objects in nature and technology can be simulated by a single rigid body. These include flight vehicles, aircrafts, spaceships, and submarines, celestial bodies with complex internal structure, and the disturbances acting on them.

This calls for studying rotational motions of the rigid and quasi-rigid (close to rigid) bodies about a fixed point under the action of external and internal perturbation torques of various physical nature. The theory and calculation methods are developed for the motion of a body containing a viscous fluid and elastic and viscoelastic elements. Such questions arise in modern problems of dynamics, orientation, control, and stabilization of the natural and artificial celestial bodies, technical objects, gyroscopy, and other fields of mechanics.

Different physical nature of the perturbation torques and their rather complex dependence on the generalized coordinates result in the equations of motion, for which precise analytical solution is hardly possible. On the other hand, the numerical solution, characterizing the particular cases in detail, does not allow tracing the general properties of the motion and its evolution. Therefore, the role of various asymptotic methods is great, which allow, if correctly applied, identifying the main features of the motion already in the first approximation.

Application of the Poincare method of small parameter for the construction of the solution of the problem, concerning the rigid body motion about a fixed point, is described in the books [9, 10]. In [11], one can find a survey of the methods for integrating the motion equations of mechanical systems.

The main purpose of the present book is to study the evolution of motion of a rigid body about its center of mass under the action of various perturbation torques. In this context, the main attention is paid to the averaging method [12–14]. This method has been long used in the celestial mechanics, though without proper substantiation. For the first time, it was strictly formulated and justified in the works of N.M. Krylov and N.N. Bogolubov [12].

Presently, there are many papers, dedicated to the substantiation and application of asymptotic methods. Exposition of these methods, as well as a detailed bibliography on this subject, is contained in the books by N.N. Bogolubov and Yu.A. Mitropolsky [12], V.M. Volosov and B.I. Morgunov [13], Yu.A. Mitropolsky [14], N.N. Moiseev [15], and V.I. Arnold, V.V. Kozlov, and A.I. Neishtadt [11, 16].

For the first time, the averaging technique was applied to studying the perturbed motions of a satellite about its center of mass in the papers of V.V. Beletsky [17] and F.L. Chernousko [18]. In [17], a satellite having a dynamic symmetry was considered. In the work [18], an averaging procedure was constructed for a satellite with an arbitrary triaxial ellipsoid of inertia, i.e., the averaging was performed over the Euler–Poinsot motion. Besides, in the paper [18], an averaging procedure was proposed for a triaxial satellite with close to one another moments of inertia. In both cases, the motion of the satellite is composed of the Euler–Poinsot motion around the vector of angular momentum and the motion of this vector itself in the space.

Let us briefly consider the works which are devoted to the research of perturbed motions of a rigid body about its center of mass and are close to the subject of the present book. The monograph [17] is dedicated to the methods of research and the main effects of the motion of an artificial satellite about its center of mass under the action of the gravitational, magnetic, aerodynamic torques and the torques due to the light pressure. In the book [19], the theory of relative motion of a satellite in the gravitational field is presented in detail, while the main attention is paid to nonlinear resonance effects. The effects of the satellite motion, described in [17, 18], were also investigated in the paper [20].

V.V. Beletsky and A.M. Yanshin [21] studied the influence of aerodynamic forces on the rotational motion of artificial satellites. The work [22] is devoted to researching the influence of decelerating aerodynamic torques on the rotational motion of satellites of various shapes. In the paper [23], the effects connected with the perturbed motion of an asymmetric artificial Earth satellite (AES) about the center of mass under the action of the forces of aerodynamic dissipation are investigated. In the work [24], the evolution of the satellite rotation is investigated using the complete formula for the dissipative aerodynamic torque.

The main research directions concerning the motion of space vehicles and simulation of the external forces acting on satellites are described in the reviews by V.M. Morozov [25], V.A. Sarychev [26], and S.K. Shrivastava and V.J. Modi [27].

The paper [28] is dedicated to the analysis of resonance effects in the rotational motion of a satellite with unequal inertia moments in the gravitational field. In the work [29], the nonresonance and resonance perturbed rotations of a triaxial satellite in the gravitational field are studied.

In the paper [30], the fast rotations of a triaxial satellite perturbed by the gravitational torque are considered. The solution expressed by elliptic functions and Jacobi integrals is valid for nonresonance rotations, provided the rotation velocity is much larger than the angular velocities of the orbital movement and precession.

Among the systems of stabilization for artificial satellites by means of the external torques, the most widespread are the systems of gravitational stabilization. The first model of gravitational stabilization of artificial satellites and the study of the dynamics of this system were presented in the work by D.Ye. Okhotsimsky and V.A. Sarychev [31]. Detailed information about the systems of gravitational stabilization can be found in the survey [26]. In the paper [32], a review is provided of the problems and works, connected with the development of passive systems of satellite orientation. A bibliography of the Russian and foreign research on the passive systems of orientation of satellites and space vehicles is presented also in the books [33–35]. In the work [36], a review is given of the basic results obtained in the applied celestial mechanics and the spacecraft motion control.

A model of dynamically nonsymmetric satellite with the moments of inertia close to each other, moving in the central gravitational field under the action of the moment of aerodynamic resistance forces, is investigated in [37]. The paper [38] is devoted to the question of evolution of the rapid motion of a satellite under the action of the gravitational and aerodynamic torques.

In the work [39], a mathematical model is proposed for the rotational motion of the "Photon" satellite. In the paper [40], the issues are considered of simulating the moments of aerodynamic forces acting on a satellite with a gravitational stabilization system.

An important field of practical applications of the rigid body dynamics is the mechanics of gyroscopic systems. Most comprehensively, the results on the mechanics of gyroscopes are reflected in the books by A. Yu. Ishlinsky [41] and K. Magnus [42]. Some cases of integration of the gyroscope motion equations in a resistant medium are considered in [6, 42–54]. The method of averaging is applied to studying the gyroscope dynamics. In the work by D.M. Klimov, G. N. Kosmodemyanskaya, and F.L. Chernousko [55], a fast motion of a heavy rigid body about a fixed point or the equivalent motion of a gyroscope with noncontact suspension is researched. Using the method of averaging, Yu. G. Martynenko continued studying the motions of gyroscopes of various types with noncontact suspension. In his book [56], he considered the motion of a conducting rigid body in the electrical and magnetic fields.

In the studies of G.G. Denisov and Yu. M. Urman [57–59], an analysis is carried out of the precession motions of a rigid body with a fixed point under the action of torques having a force function. The motion of a gyroscope with a noncontact suspension under the influence of nonconservative moments is considered in [60].

A number of studies are devoted to the dynamics of a rigid body in a resistant medium. In the works of L.D. Akulenko, D.D. Leshchenko, F.L. Chernousko, and A.L. Rachinskaya [61–64], fast rotation of a nonsymmetric heavy rigid body about a fixed point in a resistant medium is considered. The motion of the body is composed of the Euler–Poinsot motion about the vector of angular momentum (with the slowly decreasing values of angular momentum and kinetic energy) and the motion in the space of the angular momentum vector itself. As a result of the application of the averaging method, an autonomous equation is obtained, describing the motion of the vector of angular momentum. An analysis of this equation allows discovering the quasi-stationary motions, for which the motion as a whole dies out (the angular momentum and kinetic energy tend to zero), but the character of the body motion about the angular momentum vector remains invariable.

The papers of A.I. Neishtadt [65] and M.L. Pivovarov [66] study the motion about the center of mass of a nonsymmetric rigid body influenced by two small perturbation torques: a constant one in the body-fixed axes and a linear dissipative one or, alternatively, a constant one and a torque involving the terms quadratically depending on the angular velocity.

In the work [67], the perturbed motion of a rotating spacecraft on a circular orbit is considered under the action of a small aerodynamic torque proportional to the angular velocity of the body.

In the works [68–70], some analytical approximate solutions are obtained for the problem of the motion of a rigid body close to symmetrical one, as well as of a body with arbitrary inertia characteristics influenced by a moment which is constant in the body-fixed axes.

The works [71, 72] consider the problems on the motion of a heavy rigid body about a fixed point under the action of a dissipative torque, including the terms which are linear and quadratic with respect to the angular velocity.

In the paper [73] and the book [56], the motion about the center of mass is studied of a symmetric rigid body in the presence of resistance of the medium and an active rotating torque providing the constancy of the angular rotation of the rotor.

The work [74] investigates the stability of rotations of a body about the center of mass in a linearly resisting medium under the presence of a torque directed along one of its principal axes.

In the books by B. Ya. Lokshin, V.A. Privalov, V.N. Rubanovsky, V.A. Samsonov, and M.V. Shamolin [75–77], the problem of the motion of a rigid body in a resistant medium is studied. Quasi-stationary models of the medium resistance, spatial movement in a resistant medium, as well as the motion of axially symmetric bodies with a fixed point in the flow of medium are considered.

In the study [78], the Euler equations have been integrated for a symmetric gyrostat, which is considered to be a rigid body with a symmetric rotor placed inside, taking into account the external dissipative torques. The paper [79] considers a problem of the motion of an asymmetric rigid body about its center of mass in a resistant medium. A qualitative description of the phase trajectories is given; some of their characteristics and quantitative estimates are presented. In the work [80], the conditions are obtained for the global asymptotic stability of the stationary rotations of a nonsymmetric rigid body about its center of mass under the action of a constant external torque and a dissipative torque.

In the paper [81], free rotational movement is considered of a rigid body under the action of a linear viscous torque. The work [82] is devoted to the construction of an exact solution for the problem of free rotation of an axisymmetric rigid body, taking into account the viscous friction torque linearly dependent on the angular velocity of the body. In the paper [83], the evolution of the rigid body rotation under the influence of the sum of constant and dissipative moments is studied by means of numerical methods. In [84], some cases of the rigid body rotation, similar to Euler's case, under various damping torques are investigated.

The problem of rotational motion of a spacecraft under the action of light pressure forces is one of the most important parts of the dynamics of rotational motion of a rigid body about its center of mass. First, satellites and space vehicles equipped with extended solar panels or reflective antennas were studied. Then, the problems were considered of attitude control using the light pressure forces. The literature on these matters can be found in the reviews [26, 85] and the books [86–88].

In the monograph [17], rotation of a dynamically symmetric satellite on a heliocentric orbit under the action of the light pressure torque is investigated. In [89, 90], integral characteristics of the force action of the light flow on the frame, as well as the formulas for the light pressure torque, acting on a body bounded by a surface of revolution, are obtained.

One can identify the main areas of studying the influence of light pressure on the rotational motions of celestial objects. The first area is the analysis of the use of light pressure for the spacecraft orientation. In the late 1980s of the twentieth century, the studies were conducted in the Soviet Union on the astrometric project "Regatta–Astro," in the framework of which it was supposed to bring a space vehicle, oriented toward the Sun by the light radiation pressure, to a heliocentric orbit. Various aspects of the dynamics of such spacecraft were considered in the papers [91–93].

The second direction concerns the influence of light pressure on the rotational–translational motion of asteroids. Since the beginning of the 1990s, the threat of the Earth collision with a large asteroid has been actively discussed. Predicting such events requires the construction of a precise theory of the asteroid movement. In the light flow, a complex geometry of real asteroids leads to the appearance of perturbing torques, changing the orientation of the rotation axis and, as a result, changing the value of the total light pressure force, perturbing the orbital motion. As an example of this kind of studies, we mention the papers [94–96].

The third direction is the study of the so-called Yarkovsky effect. Sunlight, striking on an object, heats it up; as a result, thermal radiation appears. The impact of this effect on the movement of the center of mass was considered in the papers [97–99].

The specific features of the light pressure influence on the orientation regime and stabilization of a spacecraft with solar sails or reflective panels are studied in the book [100]. The paper [101] is devoted to calculating the principal vector and principal moment of the light pressure forces acting on a spacecraft with a solar sail.

In the works of L.D. Akulenko, D.D. Leshchenko, and A.S. Shamaev [102–104], the rotational motion is researched of a dynamically asymmetric satellite with an axially symmetric surface about its center of mass under the action of the light pressure torque. With the help of the averaging method, the evolution is studied of the rotations of a triaxial satellite, close to a dynamically spherical one, under the action of the light pressure torque in the case when the spacecraft is a body of revolution. In addition, the coefficient of the light pressure torque is approximated by trigonometric polynomials of an arbitrary order with respect to the orientation angle. A first integral is discovered for the system of averaged equations of the first approximation for the angles of nutation and proper rotation. The numerical and qualitative analysis of the phase plane is conducted; new qualitative effects of the satellite rotation are discovered.

The papers [105, 106] study the evolution of rotations of the Sun satellite, moving along an elliptical orbit with an arbitrary eccentricity under the influence of the torques of the forces of gravity and light pressure.

Let us consider the influence of the moments of internal dissipation forces on the rotation of a rigid body. The problems of the dynamics of bodies with the fluid-containing cavities are among the classical problems of mechanics. A fundamental study of the rotational motion of a rigid body having a cavity, filled with a homogeneous ideal fluid, was carried out in general formulation by N. E. Zhukovsky [107].

A great interest to the problems of rotation of rigid bodies with the fluid-containing cavities has arisen in connection with the development of the rocket and space technology. The presentation of the results on the dynamics and stability of motion about the center of mass of a body with the fluid-containing cavities is given in books by N.N. Moiseev and V.V. Rumyantsev [108], G.N. Mikishev and B.I. Rabinovich [109], and G.S. Narimanov, L.V. Dokuchaev, and I. A. Lukovsky

[110]. In the survey articles [111, 112], the formulations are presented for the problems of the stability theory and oscillations of rigid bodies with the fluid-filled cavities, various forms of the rotational motion equations and their first integrals are considered, and a systematic description is given for the results of research of the gyrostat motion.

The problems of dynamics of a rigid body with cavities, containing a viscous fluid, are significantly more difficult than in the case of ideal fluid. An important contribution to the solution of these problems has been made by the works of F.L. Chernousko [113–120], summarized in the monograph [121]. Its translation is given in [122]. These studies showed that solving the problems of dynamics of a body with a homogeneous viscous fluid can be subdivided, under some natural assumptions, into two parts—the hydrodynamic and dynamic ones—which can greatly simplify the initial problem. In the paper [113] and the first chapter of the monograph [121, 122], the results of which are used in this book, the motion is considered about the center of mass of a rigid body with a cavity filled with a fluid of high viscosity (for low Reynolds numbers). A system of ordinary differential equations is constructed there, which approximately (in the quasi-stationary approximation) describes the rotational movement of the rigid body with a fluid outside a small initial time interval, when the flow in the cavity is significantly unsteady. The influence of the fluid on the body movement is characterized, in the quasi-stationary approximation, by a tensor, which is determined only by the cavity shape. As an example, the problem is considered of the spatial motion of a free rigid body with a cavity filled with a viscous fluid.

In the works of A. I. Kobrin [123, 124], an initial period of rotation of a body with a cavity containing a high-viscosity fluid is investigated with the help of the boundary layer method, and the initial conditions for the system of equations proposed in [113, 121, 122] are specified. The controlled motion about the center of mass for a body having a cavity filled with a viscous fluid is studied.

The paper [125] is devoted to studying the stabilizing effect of a viscous fluid in a cavity on the rotation of a top around the given axis. There, on the basis of the equations obtained by F.L. Chernousko, a characteristic time of stabilization and the best orientation of the cavity relative to the rigid body are found. In [126], the oscillations on an elliptic orbit of a rigid body with a spherical cavity entirely filled with a viscous fluid are studied at low Reynolds numbers. In the papers [127, 128], the motion in a resistant medium of a rigid body with cavities filled with a fluid of high viscosity about a fixed point is considered. Fast rotational motion about the center of mass for a dynamically symmetric satellite with a cavity filled with a viscous fluid under the action of gravity torque and resistance of the medium is studied. In the work [129], a possibility is considered of damping the nutation oscillations by means of a viscous fluid filling a cavity in the rotor or within the gyroscope.

The paper [130] is devoted to the study of oscillations of a rigid body with a toroidal cavity filled with a viscous fluid. It is precisely the toroidal cavities with fluid that are used in certain systems of damping the spacecraft oscillations about the center of mass. In the work [131], the rotation is studied of a satellite with a permanent magnet in the plane of a polar elliptical orbit. The damping is investigated with the help of a viscous fluid which entirely fills the cavity of an arbitrary shape, under low Reynolds numbers. In the papers [132, 133] and the book [134], an asymptotic method is used to studying the inertial motion of a rigid body and the rotational motion of a symmetric satellite with a spherical or ellipsoidal cavity filled with a viscous fluid. Secular effects in the rotation motion of a planet, caused by the dissipation of energy in the matter of the core, are studied in [135]. According to [113, 121, 122], it is assumed that the influence of the fluid core is equivalent to the action on the "frozen" planet of a nonconservative moment of a special kind. In the works [136–138], the fast rotational motions about the center of mass of a dynamically asymmetric satellite with a cavity completely filled with a high-viscosity fluid under the influence of the gravitational and light torques are studied.

The paper [139] presents the analytical and numerical results for the rigid body with a cavity filled with a viscous fluid. The chaotic motions of a rigid body and a satellite with a cavity filled with fluid are investigated in [140–142].

In the papers [143, 144], the movement is considered relative to the center of mass for a satellite gyrostat with a fluid-containing cavity under low Reynolds numbers. A control based on the feedback principle is constructed, which stabilizes the stationary motions of the gyrostat. In the book [145], some control problems for the rotating rigid bodies with cavities filled with an ideal or a viscous fluid are considered.

A large number of works are devoted to the study of rotation of a rigid body with a movable internal mass, with elastic and dissipative elements. A survey of the studies on the mechanics of the systems of connected rigid bodies is presented in [146]. A survey of the research, published prior to the year 1980, on nonlinear dynamics of the elastic spacecraft or satellite with deformable elements is given in [147]. The works in this direction are described also in the surveys on the spacecraft dynamics [26, 27, 148, 149].

The need to consider the rotational dynamics of a system of bodies arose in connection with the development of practical astronautics. On one hand, we note the works associated with the study of the movements of satellite gyrostats, containing rotating masses, and, on the other, the works in which the elastic properties of the satellites and their components are taken into account. A number of problems in the indicated fields and a bibliography on these issues are presented in the monographs [7, 150, 151]. In the book of B.V. Rauschenbach and E.N. Tokar [152], the equations are presented for the angular motion of the carrier of a spacecraft, which contains movable masses.

The paper [153] (R.E. Roberson) considers the disturbance torques acting on a satellite, which are generated by the relative motion of the bodies inside the satellite. The work [154] (W.R. Haseltine) is devoted to the study of damping of the nutation motion of a rotating artificial Earth satellite with the help of an internal passive device. In the paper [155] (G. Colombo) and the book [156] (W.T. Thomson), the influence of the inner elasticity and dissipation on the motion of the satellite relative to its center of mass is studied.

In the space flight, there arises sometimes a necessity to suppress the chaotic rotation that occurs for one reason or another. To this end, the relative displacements of movable masses are used [157–161].

A significant number of works are devoted to the analysis of various problems of the dynamics of space vehicles containing internal movable masses. The issues of stability and instability, resonance phenomena, and the problems of control and stabilization of motions have been studied. In this regard, we can mention the works [162–180].

In the papers by F.L. Chernousko [181–183], some cases are considered of the motion of a rigid body containing movable internal masses connected to the body by means of elastic and dissipative elements. The angular motion of a body containing a mass of a continuous viscoelastic medium is investigated. A number of problems on the motion of a rigid body containing elastic and dissipative elements are examined in [184–189]. Some issues of the dynamics and stability of rotations of a rigid body containing elastic and dissipative elements were considered in the paper [190] and the books [191–194].

The monograph [195] considers the issues of the aircraft motion with large rotation angles when the deformable elements like rods, plates, or fluid masses perform oscillatory displacements under the action of inertial forces. In the book [134], stationary motions of mechanical systems with elastic elements and their stability are investigated. The monograph [196] studies the dynamics of multipiece orbiting space systems consisting of rigid and elastic deformable bodies. In the book [197], the issues connected with the movement of elastic space structures relative to the center of mass under the action of the gravitational field torques are considered. In the work [198], the transient processes related to the oscillations of an elastic satellite in its moving about its center of mass under the action of the control torque are investigated. In the paper [199], some quantitative estimates are found for the transient process, which leads a viscoelastic solid body of a spherical shape in a noncontact suspension to rotate steadily around the axis of maximum moment of inertia. The work [200] considers the free movement of a linearly elastic solid body about its center of mass.

In the papers [201, 202], a research is carried out concerning the effect of the elastic and viscous properties of bodies in their free angular motions.

The works [203, 204] study the rotational motion of a rigid body carrying viscoelastic inextensible rods. The paper [205] studies the evolution of motion of a satellite with viscoelastic flexible rods on a circular orbit. In the book [206], nonlinear oscillations of a rigid body, elastically connected with a point mass, in a central force field are studied. The research [207] is devoted to the dynamics and stability of a rigid body with internal dissipation.

The problem of evolution of the rigid body rotations about a fixed point continues to attract the attention of researchers. In the aspect of applications, the analysis of rotational motions of bodies about a fixed point is important for solving the problems of astronautics, the problems of the entry of flying vehicles into the atmosphere, and the movement of a rotating projectile and gyroscopy. Moreover, in many cases, the motion in Lagrange's case can be regarded as a generating (reference) motion of the rigid body, which takes into account the main torques acting on the body. Recall that in this case the body is assumed to have a fixed point and to be in the gravitational field, with the center of mass of the body and the fixed point both lying on the dynamic symmetry axis of the body. A restoring torque, analogous to the moment of the gravity forces, is created by the aerodynamic forces acting on the body in the gas flow. Therefore, the movements, close to Lagrange's case, have been investigated in a number of works on the aircraft dynamics, where various perturbation torques were taken into account in addition to the restoring torque. Note the papers by G.E. Kuzmak [208, 209] and V.A. Yaroshevsky [210], in which the movements relative to the center of mass of the flying vehicles entering the atmosphere at high speed were studied. The method of reference equations and the method of averaging were used.

In V.S. Aslanov's monograph [211], the motion is studied of a rotating rigid body in the atmosphere under the action of a time-dependent sinusoidal or biharmonic restoring torque and small perturbation torques.

The book [212] considers the influence on the gyroscope motion of the equatorial and axial braking torques, playing a significant role in the study of the rotational motion of artillery shells. The monograph by N.N. Moiseev [15] investigates the Lagrange problem on the motion of an axially symmetric top under the action of overturning moment, directed perpendicular to the plane passing through the symmetry axis of the top. In the paper [213], the impact on the Lagrange gyroscope motion of the direction change of the force that creates an overturning or restoring torque is considered. The work [214] investigates the influence of the dissipative forces on the stability of the permanent motions of the Lagrange gyroscope.

In the paper [215], the effect of viscous friction on the Lyapunov stability of the rotation of a heavy rigid body about a fixed point is investigated. A number of works are devoted to the study of the movement of a "sleeping" Lagrange top. In the paper [216], the sufficient conditions are found for the stability of the vertical rotation of the Lagrange top in the presence of a damping torque. In the work [217], the sufficient conditions for the asymptotic stability of a "sleeping" top in a resistant medium are obtained.

In the paper of A.M. Kovalev [218], the movement of a body, which differs little from the Lagrange gyroscope, is studied with the help of the Kolmogorov–Arnold theorem. In the work [219], the problem concerning the existence of periodic solutions for the equations of the rigid body motion about a fixed point with the distribution of mass close to Lagrange's case is considered. The book [10] describes the application to the rigid body dynamics of the averaging techniques of the Gauss type or the methods due to Fatou, N.D. Moiseev, I. Delone, and G. Hill, introduced in the celestial mechanics. The periodic motions of the Lagrange top under small displacement of its center of gravity or small deviation from its axial dynamic symmetry are studied. In the work [220], using the method of Hori, the movement is studied of a heavy rigid body with a fixed point, the mass distribution in which differs little from the case of Lagrange, whereas the center of gravity is located sufficiently close to this point. The paper [221] studies the motion of a heavy rigid body with a fixed point in Lagrange's case with an asymmetry due to inequality of the equatorial moments of inertia. The work [222] considers the origination of chaotic motions of a rigid body with a small shift of the center of mass from the dynamic symmetry axis.

In the papers [223–225], an analogy is considered between the perturbed problem of the motion of the Lagrange gyroscope in the case of potential perturbations and the problem of rotation of a satellite, the center of mass of which moves along a circular orbit in the equatorial plane, taking into account the influence of the Earth magnetic field.

In the work [226], the stationary motions of a rigid body in Lagrange's case under the action of dissipative forces and the thrust imbalance creating a restoring torque are investigated. The domains of fulfilling the conditions of stability of uniform rotations are found. The stability of the motion of symmetrical heavy rigid body in the presence of resistance forces and the engine torque with respect to the axis of symmetry, defined as a function of time, was considered earlier in [227].

In the work by V.F. Zhuravlev [228] and in the monograph [229], the problem is considered on the behavior of the Lagrange top in the case when the suspension point performs harmonic oscillations in the horizontal plane. The articles by V.N. Koshlyakov [230, 231] consider the problem of stabilization, by means of vertical vibration, of a symmetric rigid body rotating about a fixed point.

A number of other studies are dedicated to the problems of dynamics of a rigid body with a vibrating suspension point. Thus, the movement of a rapidly rotating symmetrical or close to symmetrical gyroscope under the vertical vibrations of the suspension point is investigated in the works [232, 233]. In the paper [234], the rotation of a viscoelastic solid with the movable base is considered. The work [235] investigates the perturbed angular motions of the Lagrange top under random oscillations of the fulcrum. The paper [236] considers the movement of the Lagrange top, the suspension point of which performs vertical harmonic oscillations of high frequency and small amplitude.

The work [237] studies the rotation of the Lagrange gyroscope, along the axis of dynamic symmetry of which a point mass moves under the influence of the gravity force and elastic force. In the paper [238], the influence is estimated of the moving point masses (linear oscillators), performing oscillations along the symmetry axis of the top or along the axes orthogonal to the axis of symmetry, on the stability of uniform rotation of the Lagrange top. In [239], the problem of passive stabilization of the rotations around the vertical of the Lagrange gyroscope with two degrees of freedom damper of the "swing" type is considered.

In the paper [240], the motion of a dynamically symmetric heavy rigid body with a fixed point under the action of constant and dissipative moments is considered. The stationary regimes of the system are determined and their stability is investigated. In the work [241], the effect is estimated of the dissipative and permanent torques on the stability of uniform rotation of the Lagrange top with an arbitrary axisymmetric cavity completely filled with an ideal fluid. The paper [242] is devoted to studying the motion of a symmetrical top with a cavity filled with a viscous fluid in the gravitational field, when the axis of the top is deflected from the vertical. In the work [243], the motion is under consideration of a symmetrical heavy rigid body with a fixed point under the action of the friction forces caused by the surrounding dissipative medium.

The paper [244] investigates the evolution of regular precessions of a rigid body close to Lagrange's case. In the works [245, 246], the asymptotic behavior is studied of the Lagrange gyroscope motions, close to regular precessions, under the influence of a small perturbation torque.

The work [247] offers an overview of the results obtained prior to the year 1998 on the problem of evolution of the rigid body rotations, close to Lagrange's case.

The perturbed motions of a rigid body motion, close to Lagrange's case, are investigated with the help of the averaging method in the paper of L.D. Akulenko, D.D. Leshchenko, and F.L. Chernousko [248]. It describes the conditions for the possibility of averaging the equations of motion with respect to the nutation angle; an averaged system of equations is obtained. The body motion in the medium with linear dissipation is considered. In [249], the perturbed motion of the Lagrange top under the action of the slowly time-varying linear dissipative moment is considered.

In the papers [250, 251], the perturbed fast rotations of a rigid body, close to regular precession, are considered. An averaged system of the motion equations in the first and second approximations is obtained and investigated. The works [252–254] describe the evolution of rotations in a more general case, when the value of the restoring torque depends on the nutation angle. Some examples are studied which correspond to the constant and linear external torques. The solutions of the averaged systems of equations of the first and second approximations are determined.

The paper [189] investigates the perturbed motions of a rigid body, close to the regular precession in Lagrange's case, under the influence of a slowly time-varying perturbation torque and a restoring torque depending on the nutation angle. In [255–257], the evolution is studied of the rigid body rotations, close to regular precession, under the action of a restoring torque, depending on slow time and nutation angle, as well as a perturbation torque, slowly varying in time.

In the works [258, 259], the perturbed rotational motions are considered of a rigid body, close to the regular precession in Lagrange's case, under the action of a restoring torque, depending on the angles of nutation and precession.

The presented brief survey does not purport to be complete and can be significantly expanded. However, it is clear already from this survey that there is an extensive literature on the dynamics of a rigid body, moving about its center of mass under the influence of perturbation torques of various physical nature. The research in this area is in demand in connection with the problems of motion of flying vehicles (satellites, spacecraft, aircraft, unmanned aerial vehicles), celestial bodies (planets, comets), gyroscopes, and other objects of modern technology.

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About this Book

This monograph presents the results of the authors' research on the dynamics of the rigid body motions about its center of mass. The authors consider the evolution of these motions under the influence of various perturbation torques. The basic method applied in the studies is the Krylov–Bogolubov asymptotic averaging method. These questions arise in modern problems of dynamics, orientation and stabilization of natural and artificial celestial bodies, gyroscopy and other areas of mechanics.

For all cases of motion considered in the book, we present and analyze basic equations, perform the averaging procedure and obtain the averaged equations, which, being significantly simpler than the original ones, describe the motion over a large time interval. We present the accuracy estimates for the asymptotic procedure. As a result of analysis and solution of the obtained equations, we establish some quantitative and qualitative specific features of the motions, provide a description of the evolution of the body motion. The presentation is illustrated by numerous examples.

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