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THE QUASISTATIC CONTACT PROBLEM OF THERMOELASTICITY FOR ROUGH LAYERS.

V.M.Alexandrov,[1] V.N.Gavdzinski,[2] E.V.Maltseva [3]

[1] *Moscow State University, Russia*

[2] *Odessa State Academy of Building Engineering and Architecture, Ukraine*

[3] *Odessa National Economic University, Ukraine*

Two elastic rough layers (coverings) having distinct thicknesses $h_i\left(i=1{,}2\right)$, mechanical and thermophysical characteristics are applied to undeformable backings. These bodies have been drawn together by quantity k so that the thickness of the packet of layers is equal to h_1+h_2-k. Then we assume, that $k << \min\left(h_1,\, h_2\right)$. At $t=0$ one of bodies starts to slide with respect to another in the direction of z - *axis* or x - *axis* with the velocity V. Dynamical effects are neglected. On the boundary between layers we have friction forces

$$\tau = k\left(q\right)q, \tag{1}$$

where $q\left(t\right)$ is the contact pressure changing with respect to time t slowly; $k\left(q\right)$ is the coefficient of friction depending on pressure. Friction forces give rise to wear of layers. These forces do the work

$$Q = V\tau\ , \tag{2}$$

which practically all passes into heat [1]. Therefore the problem of heat conductivity for bodies with coverings in the case of heat sources distributed in the contact region $\left(y=0\right)$ must be considered.

As far as $q\left(t\right)$ varies slowly then the process of heat conductivity in the layers may be assumed to be quasi-stationary.

We suppose that the temperature of the backing of the second layer is equal to zero and denote the temperature of the backing of the first layer by $T_0 (T_0 \geq 0)$.

$T_i^*\left(i=1,2\right)$ are the temperatures of layer surfaces in the contact region.

Solving the corresponding heat equations for layers we get

$$T_1 = T_1^*\left(1 - \frac{y}{h_1}\right) + T_0\frac{y}{h_1},$$
$$T_2 = T_2^*\left(1 + \frac{y}{h_2}\right). \qquad (3)$$

At $y = 0$ we have the condition of the imperfect contact

$$\lambda_2 T_2^* - \lambda_1 T_1^* = V\tau,$$
$$\lambda_2 T_2^* + \lambda_1 T_1^* = 2(R(q))^{-1}(T_1 - T_2), \qquad (4)$$

where $R(q)$ is the contact thermoresistance. On substituting (3) into (4) we obtain

$$T_1^* = [Vkqh_1 + (\lambda_2 R + 2h_2) + 2\lambda_1 T_0(\lambda_2 R + h_2)]\Delta^{-1},$$
$$T_2^* = [Vkqh_2(\lambda_1 R + 2h_1) + 2h_2\lambda_1 T_0]\Delta^{-1} \qquad (5)$$
$$\Delta = 2(\lambda_1\lambda_2 R + h_2\lambda_1 + h_1\lambda_2).$$

We require that T_i^* at any time don't reach the temperatures of melting of corresponding layer materials.

Therefore we must impose some restrictions on values of V and T_0.

The condition of mechanical contact between layers is

$$v_2(-h_2, t) - v_1(h_1, t) + v_2(t) - v_1(t) = k, \qquad (6)$$

where $v_1(h_1, t)$ and $v_2(-h_2, t)$ are the displacements of rigid backing in the direction of y-*axis* caused by deformation of layers; $v_1(t)$ and $v_2(t)$ are the displacements of backings in the same direction arising from wear of layers and bearing of roughness. On the basis of equations of uncoupled thermoelasticity withe regard for expressions of temperatures in layers (3) and boundary conditions

$$v_1(0, t) = v_2(0, t) = 0,$$
$$\sigma_{y_1}(0, t) = \sigma_{y_2}(0, t) = -q(t)$$

we have

$$v_1(h_1, t) = -\frac{qh_1}{G_1\delta_1} + \frac{1}{2}\beta_1 h_1(T_0 + T_1^*), \qquad (7)$$
$$v_2(-h_2, t) = \frac{qh_2}{G_2\delta_2} - \frac{1}{2}\beta_2 h_2 T_2^*,$$
$$\delta_i = 2(1 - v_1)(1 - 2v_1)^{-1},\ \beta_1 = \alpha_1(1 + v_i)(1 - v_i)^{-1},$$

where G_i and v_i are elastic constants, of layer materials, α_1 are their coefficients of linear expansion. The difference $v_2(t)-v_1(t)$ can be represented in the form [2]

$$v_2(t)-v_1(t)=V\int_0^t f(q)d\tau+qg(q) \tag{8}$$

where $f(q)$ and $g(q)$ are some non-linear functions of pressure.

On substituting (6) in (7), (8) and (5) we have the following integral equation for the contact pressure q

$$[\gamma_1(q)-V\gamma_2(q)]q-T_0\eta(q)+V\int_0^t f(q)d\tau=k, \tag{9}$$

where
$$\gamma_1(q)=h_2(G_2\delta_2)^{-1}+h_1(G_1\delta_1)^{-1}+g(q),$$
$$\gamma_2(q)=k\left[\beta_2h_2^2(\lambda_1R+2h_1)+\beta_1h_1^2(\lambda_2R+2h_2)\right](2\Delta)^{-1}, \tag{10}$$
$$\eta(q)=\left(\beta_2h_2^2\lambda_1+\beta_1h_1^2\lambda_2+2\beta_1h_1h_2\lambda_1+2\beta_1h_1\lambda_1\lambda_2R\right)\Delta^{-1}.$$

The functions $k(q), R(q), f(q)$ and $g(q)$ are often taken in the form

$$k=k_0\left(Aq^{\alpha}+1+Bq^{-\beta}\right), R=Cq^{-\gamma},\ f=Dq^{\delta},\ g=Eq^{\varepsilon},$$

where all constants are defined by experiment and

$$\alpha<1,\ \beta\le 1, \gamma=1, \delta\ge 1, -1<\varepsilon\le 0. \tag{11}$$

The integral equation (9) is equivalent to the differential equation

$$\left[\gamma_1(q)-V\gamma_2(q)-V\gamma_2'(q)q-T_0\eta'(q)q+Vf(q)\right]=0 \tag{12}$$

and the initial condition

$$\{[\gamma_1(q)-V\gamma_2(q)]q-T_0\eta(q)\}_{t=0}=k \tag{13}$$

The latter in general is a transcendental equation with respect to $q(0)$.

The existence of the solution of this equation when $q(0)>0$ is the condition of thermal stability [4]. Therefore V and T_0 must be subjected to one more restriction.

The fulfillment of the above mentioned condition ensures the damping of $q(t)$ when $t\to\infty$. Having found $q(0)$ from the equation (13) we can obtain the solution of the equation (12) and the contact layer temperatures by formulas (5).

We consider the special ease.

Let $A=B=\alpha=\beta=\varepsilon=0, \delta=\gamma=1.$

Then the equation (9) takes the form

$$\left(\gamma_1 - V\frac{a+bq}{c+dq}\right)q + VD\int_0^t q(\tau)d\tau = k + T_0\frac{m+nq}{c+dq}, \qquad (14)$$

where

$$a = 2^{-1}k_0C\left(\beta_2h_2^2\lambda_1 + \beta_1h_1^2\lambda_2\right), \qquad b = k_0h_1h_2\left(\beta_2h_2 - \beta_1h_1\right),$$
$$c = 2\lambda_1\lambda_2C, \qquad d = 2\left(h_2\lambda_1 + h_1\lambda_2\right), \qquad m = 2\beta_1h_1\lambda_1\lambda_2C, \qquad (15)$$
$$n = \beta_2h_2^2\lambda_1 + 2\beta_1h_1h_2\lambda_1 + \beta_1h_1^2\lambda_2.$$

We construct the asymptotic solution of the equation (14) for small values of time. To this end we seek the solution $q(t)$ in the form of the expansion

$$q(t) = \sum_{i=0}^{\infty} a_i t^i. \qquad (16)$$

Substituting (16) in the equation (14) and equating coefficients of like powers of t we obtain the following relations for the determination of the first three a_i

$$a_0\left(\gamma_1 - \frac{VA_*}{C_*}\right) - \frac{T_0M_*}{C_*} = k,\ Pa_1 + VDa_0 = 0,$$
$$Pa_0 + \frac{1}{2}VDa_1 - \frac{a_1^2V}{C_*}\left(b - \frac{A_*d}{C_*}\right)\left(1 - \frac{da_0}{C_*}\right) + \frac{T_0a_1^2d}{C_*^2}\left(n - \frac{M_*d}{C_*}\right) = 0, \qquad (17)$$
$$A_* = a + ba_0,\ C_* = c + da_0,\ M_* = m + na_0,$$
$$P = \gamma_1 - \frac{VA_*}{C_*} - \frac{Va_0}{C_*}\left(-\frac{A_*d}{C_*}\right) - \frac{T_0}{C_*}\left(n - \frac{M_*d}{C_*}\right).$$

Here the first relation is the non-liner algebraic equation with respect to a_0. The unique solution of this equation such that $a_0 > 0$ exists under the conditions

$$\gamma_1 d > Vb, \qquad \gamma_1 c > Va + T_0n + kd. \qquad (18)$$

These conditions are the conditions of thermal stability.

The second and the third conditions (17) are used for successive determination of a_1 and a_2. The possibility of constructing of subsequent terms $(i \geq 4)$ of the asymptotic solution (16) with simultaneous redetermination of previous terms is obvious. Now we construct the asymptotic solution of the equation (14) for large values of t. To do this we reduce the equation (14) to the following equation

$$[F + Gql(q)]q + VD\int_0^t q(\tau)d\tau = H, \qquad (19)$$

where

$$F = \gamma_1 - \frac{Va}{c} + \frac{T_0}{c}\left(\frac{md}{c} - n\right), \qquad H = k + \frac{T_0 m}{c},$$

$$G = \frac{V}{c}\left(\frac{ad}{c} - b\right) - \frac{T_0 d}{c^2}\left(\frac{md}{c} - n\right), \quad l(q) = \left(1 + \frac{d}{c}q\right)^{-1}.$$

Note, that $F > 0$ by virtue of the second condition (18). The quantity G may be positive or negative depending on parameters of the problem.

We solve the equation (19) by method of successive approximations with regard for the fact that $q \to 0$ when $t \to \infty$. Namely, the first approximations we hind from the equation (19) when the second term in the square bracket is discarded.

It is easily verified that

$$q = \lambda e^{-\mu t}, \lambda = HF^{-1}, \mu = VDF^{-1} \qquad (20)$$

The second approximation we find from the equation

$$(F + Gq_1)q_2 + VD\int_0^t q_2 d\tau = H \qquad (21)$$

We reduce this equation to the equivalent differential equation with the corresponding initial condition. Next we have

$$q_2 = E(F + G\lambda)^{2G\lambda - 1}\left(F + G\lambda e^{-\mu t}\right)^{-2G\lambda} e^{-\mu t}. \qquad (22)$$

The possibility of constructing of subsequent approximations for the solution of this equation for values of t is evident.

Then the contact temperatures T_1^* and T_2^* for small and large values of time can be found by formulas (16), (22), and (5).

Let us take note of the fact that the contact temperatures are maximal when $t = 0$. This is the defect of the qusi-stationary statement of the problem. However the contact temperatures reach maximal values for very small relative time. Then they begin to diminish to T_0 and zero slowly. This occurs due to the fact that $q(t)$ tends to zero and the contact thermoresistance tends to infinity.

Conclusion

When one solid body is slid on another, the temperature in the vicinity of the interface will tend to rise smoothly with time. Wear of the surfaces has been associated with thermal stresses. The appearance of such a disturbance has been attributed to a kind of instability where the material in the vicinity of a heated asperity expands and relieves the surrounding region of load, thereby increasing the heating of the region.

Summary

The quasi-static plane problem of uncoupled theory of thermo elasticity for rough coverings of rigid bodies with regard for friction heating and wear are considered. Non linear Volterra’s equation for the pressure is obtained. In special case the asymptotic solutions of this equation for small and large relative time are found. The phenomenon of the thermal instability is described and examined.

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