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## LAMB WAVE MODE DISPERSION IN COMPOSITE LAMINATES

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**Abstract.** This study aims to improve the in situ structural health monitoring method to localize the impact source and determine the Lamb flexural mode A0 velocity in composite structures with unknown stacking and cross-section. The algorithm is based on the differences in stress waves that can be detected by a finite number of sensors and is divided into two stages. In the first stage, the squared modulus of the continuous wavelet transform, which ensures high accuracy of the time-frequency analysis of acoustic waves, was used to determine the arrival time of the Lamb flexural wave. In the subsequent step, the spatial coordinates of the impact location are determined, as well as the magnitude of the group velocity. The coordinates and velocity can be determined by solving a set of nonlinear equations using a combination of the local iterative Newton method coupled with the line search and polynomial backtracking methods. The proposed method, in contrast to current impact localization algorithms, does not require a priori knowledge of the angular group velocity anisotropy of the measured waveforms, as well as the mechanical properties of the structure. The results showed that the source location was achieved with satisfactory accuracy (the maximum error in the impact location estimate did not exceed 3% of the laminated composite sample dimensions).

**Key words:** impact area localization, laminated composites, continuous wavelet transform, Lamb waves

**Introduction.**

In isotropic or quasi-isotropic materials, impact source location is usually performed using the time-of-arrival triangulation method (also known as the Tobias algorithm) [1, 2]. However, since the group velocity in isotropic media is assumed to be constant in all directions, these methods are not suitable for anisotropic and inhomogeneous structures, which include laminated composite structures. Recently, a number of studies presented in the literature have focused on the detection and location of the acoustic emission source (impact event) in composite materials [3].

Different methods have been used to spatially localize the inhomogeneities of the impact field. Direct processing of triangulation measurements in dispersive media was often supplemented by wavelet transforms, in particular, by the modification of Lamb waves.

One of the most widely used models included a model method for reconstructing the force history and identifying the impact location [4]. This methodology was based on minimizing the difference between the actual and predicted response under local action on the volume of laminated composite. The method of minimizing the difference between the actual and predicted response can be used for any kind of anisotropic material, even with complex geometry. However, the disadvantages of this method include the need to know the mechanical properties of the medium and a theoretical model to simulate the dynamic-acoustic behavior of the structure.

Among others, it is worth mentioning the algorithm for identifying the impact point, which assumes knowledge of the elliptical pattern of the angular group velocity.

This method requires a priori knowledge of the group velocities at  $0^\circ$  and  $90^\circ$  relative to a planar  $x$ - $y$  reference frame. Therefore, such a method can only be applied to quasi-isotropic and unidirectional composite structures. An alternative approach is a technique consisting of minimizing the error function representing the differences in the recorded triangulation signals. In this case, nonlinear least-squares optimization methods were used to calculate the stiffness of the laminated plate from the measured group velocities.

The aim of this work is to modify the previous study to localize the impact region in the volume of laminar composite and the magnitude of the group velocity using a finite number of recorders. The magnitude of the group velocity is determined using the Lamb bending mode A0 in composite plate structures.

### Impact localization methodology.

Determining the location of the impact source is a mathematical inverse problem. Consider the point of the impact source  $I$ , which is located at unknown coordinates  $(x_i, y_i)$  in the plane of the plate  $x$ - $y$ . The transducers are located at a distance  $l_i$  from the source, and  $d_{km}$  are the distances between each pair of transducers  $k$  and  $m$ . In addition, the dimensions of the plate are  $L$ , the length, and  $W$ , the width. The coordinates of the acoustic emission source can be determined by solving the following equations

$$\|I_i\|^2 = (x_i - x_l)^2 + (y_i - y_l)^2, \quad t_i = \|l_i\|/V_{gi}, \quad (1)$$

where  $V_{gl}$  is the velocity propagation of the stress wave reaching the  $i$ th transducer;  $t_i$  is the time of detection of acoustic signals;  $(x_i, y_i)$  are the coordinates of the  $i$ th sensor.

The dispersive nature of the Lamb bending mode and the uncertainty of the acoustic signal level can significantly reduce the level of source localization. Therefore, a good impact detection method requires a suitable choice of time-frequency analysis for the acoustic triangulation method. The wavelet transform method provides a good compromise between time and frequency resolution, and it is capable of analyzing low and high frequencies simultaneously. The continuous wavelet transform is linear and correlates the harmonic waveform  $u(x, t)$  with the basis functions for the mother wavelet  $c(t)$

$$WT(x, a, b) = (a)^{-0.5} \int_{-\infty}^{+\infty} u(x, t) \psi^* \left( \frac{t-b}{a} \right) dt, \quad (2)$$

where  $\psi^*(t)$  denotes the complex conjugate of the mother wavelet  $\psi(t)$ ,  $a$  is the dilatation of scale parameter,  $b$  is the translation parameter.

The mother wavelet function is expressed by the following equation

$$\psi(t) = (\pi F_b)^{-0.5} \exp(-t^2 / F_b) \cdot [\cos(\omega_c t) + j \sin(\omega_c t)], \quad (3)$$

where  $f_c = \omega_c / 2\pi$  is the central frequency,  $F_b$  is the shape control parameter (wavelet bandth).

The waveforms were analyzed with respect to the group (energy) velocity, which was defined as the velocity of the modulated wave. The modulated wave consisted of two waves of unit amplitude with slightly different frequencies  $\omega_1$  and  $\omega_2$ , which propagated in the  $x$ -direction of the thin plate as follows

$$u(x, t) = \exp[-j(k_1 x - \omega_1 t)] + \exp[-j(k_2 x - \omega_2 t)], \quad (4)$$

where  $k_1$  and  $k_2$  are the wave numbers.

### Summary and conclusions.

In this research work structural health monitoring method is presented for localizing the acoustic signal source (impact event) and for determining the elastic wave velocity in plate-like composite structures. The proposed method is based on the wavelet analysis of the stress wave difference, which can be measured by a finite number of sensors attached to the surface. The peak value of the scalogram was used to identify the triangulated acoustic signal of the Lamb A0 flexural mode. The coordinates of the impact location and the group velocity were obtained by solving a set of nonlinear equations using a combination of the local iterative Newton method coupled with global unconstrained optimization (line search and polynomial backtracking method). Analysis of the calculation results for laminated composites showed that the identification of the source location and the numerical determination of the Lamb wave group velocity can be determined with satisfactory accuracy.

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