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MULTISCALE HOMOGENIZATION OF REINFORCED COMPOSITES

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One of the most effective approaches to computational modeling of composite systems is the homogenization method [1]. A necessary condition for applying the homogenization method is the presence of a certain scale relationship between the components of the reinforced composite and the entire system [2]. Most often, two-scale analytical models are introduced, each of which is associated with a pre-set scale parameter. To implement the effective computational processes, this parameter is specified as a small value, namely, a real number (usually tending to zero). The impossibility of introducing more than two different scales for a local volume of a composite and, in addition, the insufficient sensitivity of the homogenized characteristics of the composite to the geometric relationships of scales can be attributed to the significant disadvantages of such methods [3].

Wavelet analysis techniques can implement the multiscale ideology. Such a paradigm is a very modern and widely developed numerical method in signal theory, and, most importantly, performs the analysis of composite systems with several geometric scales. The multiscale technique can be considered as a more realistic analysis for most engineering composites, which allows for simultaneous analysis for different scales of microdefects, interface, reinforcement and the entire structure. It should be noted that wavelet analysis is a particularly promising tool in the field of reinforced composite materials, where it allows two goals to be achieved. The first goal is the possibility of constructing multiscale heterogeneous structures using fixed wavelets, which brings the description as close as possible to the real production process. Achieving this first goal allows for the analysis of experimental results on the morphology of the composite. The second goal is related to the multidimensional decomposition of the spatial distribution of the composite material and its physical properties [4]. In this case, wavelets of different types are used for different scales. The multiscale strategy in this study is implemented to reduce and homogenize linear and unidirectional problems. For this purpose, a bounded linear operator $S_j : V_i \to V_j$ is used. The quantity V_j is covered by the translations of the function $\varphi(2^j x - k, \lambda, p, q), \lambda = const$, where *p* and *q* are vector-valued forcing terms. Therefore, this quantity can be represented as a matrix. The matrix is finite provided that the multi-resonator analysis is defined on a limited domain. Next step in multi-scale analysis is to use the relation

$$S_j x = f. \tag{1}$$

The decomposition of $V_j = V_{j+1} \oplus W_{j+1}$ leads to a partition of S_j into four parts and, therefore, to the definition of the wavelet space W_{j+1}, which is the detailed or small-scale component of V_j) using the matrix equation

$$\begin{pmatrix} A_{sj} & B_{sj} \\ C_{sj} & T_{sj} \end{pmatrix} \begin{pmatrix} d_x \\ s_x \end{pmatrix} = \begin{pmatrix} d_f \\ s_f \end{pmatrix}, \tag{2}$$

where:

$A_{sj} : W_j \to W_{j+1}, B_{sj} : V_{j+1} \to W_{j+1}, \ A_{sj} : W_j \to V_{j+1}, \ T_{sj} : V_j \to V_{j+1};$

$d_x, d_y \in W_{j+1}, s_x, s_y \in V_{j+1}.$

The multiscale homogenization procedure with parameter *K* was written as

$$Bx + q + \lambda = K(Ax + p), \tag{3}$$

$$B_j = diag\left\{I + (B_j)_i\right\}_{i=0}^{i=2^j-1}, \tag{4}$$

$$A_j = diag\left\{I + (A_j)_i\right\}_{i=0}^{i=2^j-1}. \tag{5}$$

The recurrence relations are local and can be carried out at as many scales as necessary. In the case of this work |the calculation method was extended for multi-resolution coefficients up to 20. The analysis showed significant nonlinearity of both the real and imaginary parts of the homogenization parameter *K*. The main advantage of this calculation method is that this procedure allows the homogenization coefficients to vary in an arbitrary number of intermediate scales. This contrasts with classical homogenization examples that did not allow any intermediate scales. A general framework is constructed for multiscale reduction and homogenization. The multi-wavelet basis is implemented using the Haar-wavelet for systems of linear ordinary differential equations. Since the Haar-functions at a fixed scale have no overlapping supports, the recurrence relations for the operators and boost terms in the equation can be written as local relations and solved explicitly.

Summary and conclusions. The consequence of the application of the wavelet multiscale method was the calculation of probability moments of real and imaginary surfaces of effective homogenization parameters. As a promising direction, it is necessary to consider the possibility of determining the relations for homogenized coefficients in terms of volume fractions of layers, as well as the extension of the homogenization method to inhomogeneous multiscale media with a more general periodic geometry. The entire methodology can be adopted without any changes in stochastic reliability studies, where probability coefficients of effective properties or state functions can be used in calculations of the reliability index.

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