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INTRODUCTORY PHYSICS COURSE FOR THE PREPARATORY DEPARTMENT

MECHANICS
MOLECULAR PHYSICS

Odessa – 2023

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The textbook covers the basic laws, formulas and examples of problem solving for courses in mechanics and molecular physics. Each section contains the required number of test tasks for students to solve independently. The proposed educational material can be used for foreign students of the preparatory department.

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PREFACE

The course of physics plays the role of a basic component of the natural science education of students of higher educational institutes. The educational work of foreign students of the preparatory department for the study of the physics course consists of the following main elements: studying physics according to textbooks, solving problems, performing control tasks, laboratory tasks and passing the exam. The manual describes the basic laws and phenomena within the scope of the school physics course. Each section contains a brief summary of the theoretical material, methodological instructions for solving problems, examples of solving typical problems, the form of their recording and the necessary explanations. All physical quantities included in the problem are given in SI. The manual contains problems and test tasks for self-solving, requirements for the design of problem solutions, approximate calculations and rounding rules.

The text is lexically adapted taking into account the students' language difficulties and the experience of teaching a physics course to foreign students. The manual contains simple and complex combined tasks intended for different levels of basic training of foreign students.

The textbook is designed for classroom studies and independent work of preparatory department's foreign students. However, it can be used to prepare college students for external independent assessment in physics.

PRELIMINARY INFORMATION

Curriculum of discipline "Physics"

The requirements for the preparation of students suggest that as a result of studying the sections of the discipline "Physics" it is necessary to learn how to solve problems on the following topics of the curriculum:

MECHANICS

1. Kinematics. Kinematics of a material point. Material point. Average velocity. Instantaneous velocity. Rectilinear uniform motion. Rectilinear uniformly accelerated motion. Acceleration. Acceleration components: normal and tangential acceleration. Movement of a body vertically under the influence of gravity. Motion of a body thrown horizontally, motion of a body thrown at an angle to the horizon. Uniform circular movement.

2. Fundamentals of dynamics. Force. Resultant of all forces. Body mass. Newton's laws. Forces in mechanics.

3. Conservation laws. Body momentum. Momentum conservation law. Mechanical work. Power. Types of mechanical energy. Law of energy conservation.

4. Mechanical oscillations. Harmonic oscillations. Elastic pendulum. Mathematical pendulum.

5. Special theory of relativity.

MOLECULAR PHYSICS

6. Molecular-kinetic theory of an ideal gas. Basic equation of the molecular kinetic theory of gases. Ideal gas laws. Properties of steam. Air humidity. Properties of liquids. Capillary phenomena.

7. Thermodynamics. Ideal gas internal energy**.** Work done by an ideal gas. Heat. Heat balance equation**.** First law of thermodynamics. Carnot cycle.

Methodical guidelines for problem solving

Solutions to problems must be accompanied by short but comprehensive explanations with the obligatory use of drawings. Students should be guided by the following general rules when solving problems.

1. It is necessary to understand what phenomena or processes occur according to the condition of the problem.
2. Students must understand what laws determine these processes and phenomena.
3. It is necessary to understand the content of physical quantities that describe these processes and are included in the formulas of the corresponding laws.
4. Before solving the problem, it is necessary to make a diagram on which the corresponding values and directions of vector values will be indicated.
5. It is necessary to find out which values, which are included in the selected formulas of the laws, are given and which should be determined. In addition, it is highly desirable to find the table values necessary for calculations in the reference literature.
6. Students must consistently use formulas for finding unknown quantities, and obtain final working calculation formulas for the quantities to be determined. Problems should be solved in a general way. Numerical values are substituted only in the final formulas expressing the sought quantities.
7. Numerical calculations must be performed in the SI system.
8. After solving the problem, you should make sure that the general formula will give the correct dimension (unit) of the desired value. To do this, the dimension of all quantities should be substituted into the formula and the necessary actions should be performed. If the dimension obtained in this way does not match the dimension of the desired value, the solution is incorrect.

Requirements for the design of problem solving

The solution of the problem should be formalized in accordance with the following requirements.

1. Give a short condition, translating, if necessary, the numerical values of the quantities into the SI system.
2. It is necessary to provide additional tabular data to solve the problem.
3. If necessary, draw a diagram.
4. The solution to the problem should be accompanied by concise comments:

a) indicate which phenomenon or process occurs in the problem;

b) Justify the application of relevant laws and formulate them;

c) explain all notations, i.e. indicate each quantity included in the formula, if necessary, explain why this quantity has one or another sign (“+” or “-”), and a vector quantity has a specific direction.

5. Students must fully present the process of obtaining calculation formulas, including mathematical, algebraic calculations or geometric drawings, etc
6. After obtaining the general formulas, it is necessary to perform a calculation (in the SI system or another, but one system of units), check the dimensions, and evaluate the physical reality of the results.
7. At the end of the task, it is necessary to formulate a complete answer.

Approximate calculations and rounding rules

The solution of physical problems usually involves working with approximate numerical values of quantities, so the result of the calculations will also be approximate and should be rounded correctly.

Rules for calculating numerical data when performing mathematical operations.

1. When adding, subtracting, multiplying, and dividing, as many significant digits are kept in the answer as there are in the number with the smallest number of digits.
2. The result of calculating the values of a function of some approximate number must contain as many significant digits as there are in the given number.
3. When calculating intermediate results, it is necessary to store one significant digit more than rules 1 and 2 recommend (the so-called spare digit). Ultimately, the spare digit is eliminated while respecting the rounding rules.

Rounding rules

1. If the first of the discarded digits (counting from left to right) is greater than 5, then the last digit that is retained is increased by one.
2. If the first digit to be discarded (counting from left to right) is less than 5, then the last digit to be retained is not changed.
3. If the first of the discarded digits is 5, but is the result of preliminary rounding, then the rounding depends on how the first of the discarded digits is rounded:

a) when it is rounded up (e.g.: 0.165 is obtained by rounding 0.1648), the last stored digit does not change $0{,}165 \approx 0{,}16$.

b) when it is rounded down (e.g. 0.135 obtained by rounding 0.1352), the remaining last digit is increased by one: $0{,}135 \approx 0{,}14$.

4. If only one digit 5 is discarded, and there are no significant digits after it, then rounding is done to the nearest even number, that is, the last of the digits stored remains unchanged if it is even, and is increased by one if it is odd.

Operations with vector quantities

Vector quantities are characterized by a numerical value, a direction and a point of attachment. The numerical value of a vector is called its modulus. The modulus of a vector is a scalar, and it is always positive. A scalar value is characterized by only one numerical value. The modulus of vector $\vec{A}$ is denoted by $|\vec{A}|$.

The rules for adding and subtracting vectors are illustrated in Fig.1.

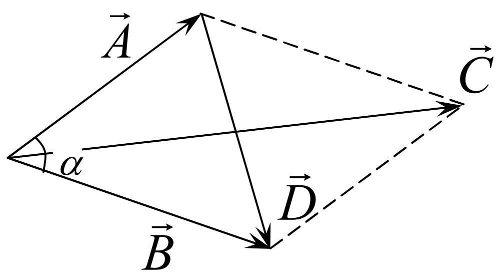


Figure 1

The following relations are true: $\vec{C} = \vec{A} + \vec{B}$ and $\vec{B} - \vec{A} = \vec{D}$:

$$|\vec{C}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}| \cdot |\vec{B}| \cdot \cos\alpha},$$

$$|\vec{D}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}| \cdot |\vec{B}| \cdot \cos\alpha}.$$

The projection of a point is the point obtained by the intersection of the normal restored from the point to the axis l with this axis.

The projection of a vector $\overrightarrow{AB}$ onto an axis oX is called a quantity

$a_x = |\overrightarrow{AB}| \cos\alpha$, where α – is the angle between the direction of the vector and the axis OX

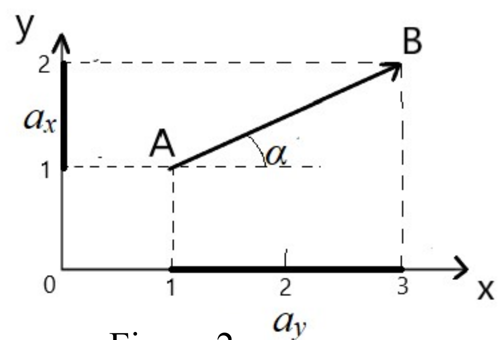


Figure 2

(because $\overrightarrow{AB}$ is a free vector, it can be positioned using a parallel transfer so that the line along which it is directed crosses the axis OX. The projection of the vector $\overrightarrow{AB}$ on the axis OY is called the value $a_y = |\overrightarrow{AB}| \sin\alpha$, where α is the angle between the direction of the vector and the axis l ($\vec{A}$ is a free vector, and it can be positioned using a parallel transfer so that the line along which it is directed crosses the axis l). The projection has a positive sign if the projection of the end of the vector is on the side of the end of the vector in which the direction of the axis l points, and a negative sign otherwise.

MECHANICS

1. KINEMATICS

Kinematics is a branch of mechanics that studies the mechanical motion of bodies without considering the causes that determine this motion.

Material point is a body whose dimensions in this problem can be neglected and it can be assumed that the entire mass of the body is concentrated in one point.

Trajectory of movement is a line along which a material point moves. Depending on the shape of the trajectory, the movement can be straight or curved.

Path l is the length of the trajectory.

Displacement $\vec{S}$ is a vector drawn from the initial position to the final position (Fig.3.)

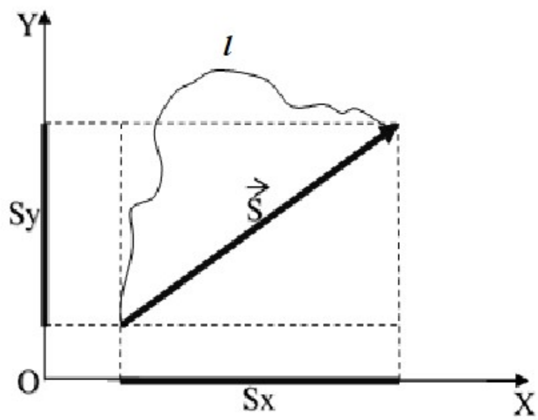


Figure 3

The length of this vector is the displacement module, and its projections on the coordinate axis are equal S_x and S_y. The length of the displacement vector can be calculated by its projections or by coordinates according to the Pythagorean theorem

$$S = \sqrt{S_x^2 + S_y^2},$$

where $S_x = x - x_0$, $S_y = y - y_0$. x_0 y_0 are the coordinates of the body at the moment of the start of the time countdown (initial coordinates).

1.1. Uniform rectilinear movement

Uniform rectilinear motion is such a mechanical motion during which the body makes the same movements for any equal time intervals. In rectilinear motion, the modulus of the displacement vector is equal to the path:

$$\left|\vec{S}\right| = l.$$

Instantaneous velocity of movement of the body $\vec{\upsilon}$ is the velocity of movement of the body at a given moment of time and at a given point of the trajectory

The SI unit of velocity is a meter per second (m/s): [υ] = m/s.

The projection of the instantaneous velocity onto the OX axis is equal to the derivative of the coordinate or the projection of the body's displacement over time

$$\upsilon_x = x'(t) = S_x'(t).$$

The **average velocity of body movement** $\vec{\upsilon}_a$ is a vector value that is equal to the ratio of the movement of the body to the time interval during which this movement is carried out:

$$\vec{\upsilon}_a = \frac{\vec{S}}{t}.$$

The **velocity** of uniform rectilinear motion does not change over time and is determined by the formula:

$$\vec{\upsilon} = \frac{\vec{S}}{t}.$$

The **modulus of the velocity** of uniform rectilinear motion does not change with time and is determined by the formula:

$$\upsilon = \frac{S}{t}.$$

The **projection of the velocity** of uniform rectilinear motion does not change over time and is determined by the formula:

$$\upsilon_x = \frac{S_x}{t}.$$

The displacement projection equation has the form $S_x = \upsilon_x t$.

Since $S_x = x - x_0$, then $x - x_0 = \upsilon_x t$.

Coordinate equation (body motion equation)

$$x = x_0 + \upsilon_x t,$$

where x_0 is the coordinate of the body at the moment of the beginning of the time countdown (initial coordinate).

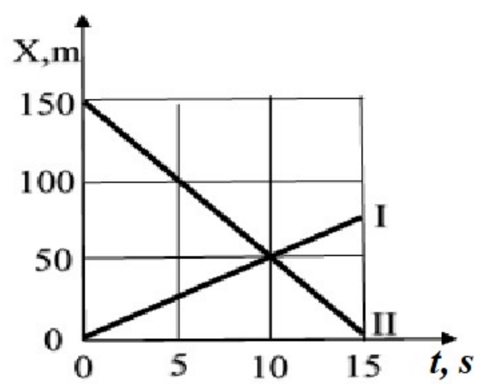
The **path** is equal to the displacement module

$$l = s = \upsilon t.$$

Problem-solving examples

1. The motion of two cyclists is given by the equations: $x_1 = 5t$ and $x_2 = 150 - 10t$. Build a dependency graph $x(t)$. Determine how long the meeting will take place and the coordinates of this point.

Known quantities:
$x_1 = 5t$, $x_2 = 150 - 10t$
$x, t, x(t) - ?$



Problem solution

At the moment of meeting, the coordinates will be the same, i.e. $x_1 = x_2$. $\Rightarrow$

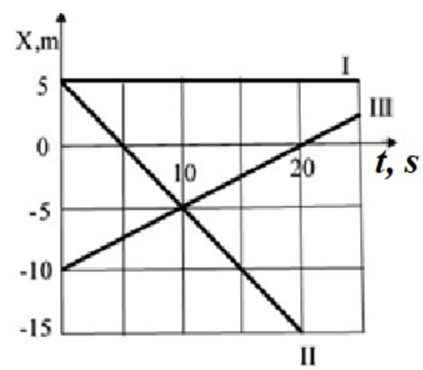
$$5t = 150 - 10t.$$

Hence the meeting time is $t = 10s$. Coordinate of the meeting point is $x_1 = 5 \cdot 10 = 50\,m$.

Answer: $t = 10\,s$, $x_1 = 50\,m$

2. The figure shows graphs of the movement of bodies I, II, III. Write the equation of motion of bodies $x(t)$. Find the time and place of meeting of the bodies.

Known quantities:
motion graphics
$x - ?\ t - ?\ x(t) - ?$



Problem solution

Since the movement of bodies is rectilinear, the dependence of the coordinate on time has the form:

$$x = x_0 + \upsilon_x t.$$

For body I, the coordinate does not change over time $x_{01} = 5\,m$, therefore the speed is zero $\upsilon_1 = 0$. For body II, the velocity can be found from the equation: $\upsilon = \dfrac{x - x_0}{t}$, where $x_{02} = 5\,m$ and $x_2 = -15m \Rightarrow \upsilon_2 = \dfrac{-15-5}{20} = -1\,m/s$.

For body III $x_{03} = -10m$, $x_3 = 0m$, $\upsilon_3 = \dfrac{0+10}{20} = 0{,}5\,m/s$.

Equations of motion of bodies: For body I: $x_1 = 5m$.

For body II: $x_2 = 5 - t$.

For body III: $x_3 = 0,5t - 10$.

We will find the meeting time by equating the coordinates $x_1 = x_2$,

$$5 - t = 0,5t - 10 \Rightarrow t = \frac{15}{1,5} = 10s.$$

Meeting point: $x = 5 - 10 = -5m$.

<u>Answer:</u> $x_1 = 5m$; $x_2 = 5 - t$; $x_3 = 0,5t - 10$; $t = 10c$; $x = -5m$.

3. The car traveled the first half of the way at a speed of $\upsilon_1 = 50 km/h$, and the second half at a speed of $\upsilon_2 = 70 km/h$. Determine the average speed along the entire path.

<u>Known quantities:</u>	*<u>Problem solution</u>*
$S_1 = S_2 = S$ $\upsilon_1 = 50 km/h = 13,9 m/s$ $\upsilon_2 = 70 km/h = 19,4 m/s$	Let's denote the first and second half of the path S_1 and S_2, that is, all the way $S_1 + S_2 = 2S$.
$\bar{\upsilon} - ?$	

The average speed of the car is determined by the formula

$$\bar{\upsilon} = \frac{S_1 + S_2}{t_1 + t_2} = \frac{2S}{t_1 + t_2}.$$

The time of movement of the first half of the path is: $t_1 = \frac{S_1}{\upsilon_1} = \frac{S}{\upsilon_1}$.

The time of movement of the second half of the path is: $t_2 = \frac{S_2}{\upsilon_2} = \frac{S}{\upsilon_2}$,

$$\bar{\upsilon} = \frac{2S}{\frac{S}{\upsilon_1} + \frac{S}{\upsilon_2}} = \frac{2\upsilon_1\upsilon_2}{\upsilon_1 + \upsilon_2}, \quad \bar{\upsilon} = \frac{2 \cdot 13,9 \cdot 19,4}{33,3} = 16,2 m/s.$$

Answer: $\bar{\upsilon} = 16,2\, m/s$.

1.2. Uniformly accelerated rectilinear motion

Uniformly accelerated rectilinear motion is motion in which the speed of motion changes equally over any equal time intervals.

Acceleration of body movement $\vec{a}$ is a vector value that characterizes the rate of change of body movement speed. The SI unit of body acceleration is meter per second squared, m/s^2: $[a] = \frac{m}{s^2}$.

For uniformly accelerated body movement:

$$\vec{a} = \frac{\vec{\upsilon} - \vec{\upsilon}_0}{\Delta t}, \qquad a_x = \frac{\upsilon_x - \upsilon_{0x}}{\Delta t},$$

where $\vec{\upsilon}$ – is the speed of movement of the body over a time interval Δt; $\vec{\upsilon}_0$ is the initial velocity of the body; a_x, υ_x and υ_{0x} are the projections of acceleration, velocity, and initial velocity of body movement on the OX axis, respectively.

The projection of the instantaneous speed is determined by the formula $\upsilon_x = \upsilon_{0x} + a_x t$.

The displacement projection equation is determined by the formula:

$$S_x = \upsilon_{0x} t + \frac{a_x t^2}{2} \quad \text{or} \quad S_x = \frac{\upsilon_x^2 - \upsilon_{0x}^2}{2a}.$$

Coordinate equation (equation of body motion):

$$x = x_0 + \upsilon_{0x} t + \frac{a_x t^2}{2}.$$

Movement of a body vertically under the influence of gravity

Free fall is the movement of a body only under the influence of gravity, the fall of a body in a windless space. During free fall, all bodies fall to Earth with the same acceleration, which is called the **acceleration of free fall**. Near the Earth's surface, the acceleration of free fall is approximately equal to 9,81 m/s^2.

The equation of free fall

1) $\upsilon_0 \neq 0;$

$$\upsilon = \upsilon_0 + gt,$$

$$h = \upsilon_0 t + \frac{gt^2}{2},$$

$$\upsilon = \sqrt{2gh + \upsilon_0^2},$$

$$y = y_0 + \upsilon_{0y} t + \frac{gt^2}{2};$$

2) $\upsilon_0 = 0;$

$$\upsilon = gt,$$

$$h = \frac{gt^2}{2},$$

$$\upsilon = \sqrt{2gh},$$

$$y = y_0 + \frac{gt^2}{2}.$$

If a body is thrown vertically upwards, then $\upsilon = \upsilon_0 - gt,\ h = h_0 + \upsilon_0 t - \frac{gt^2}{2}.$

Problem-solving examples

1. The body, which was thrown vertically upwards, returned to the ground after a time of 3s. Find the initial velocity of the body υ_0 and the height h, to which it rose. Consider that the beginning of the time countdown is zero.

Known quantities:

$\Delta t = t = 3s$

$\upsilon_0 - ?,\ h - ?$

Problem solution

Let's direct axis y vertically upwards. Equation of kinematics in projections on this axis: $y(t) = \upsilon_0 - \frac{gt^2}{2}$, $\upsilon(t) = \upsilon_0 - gt$. At the maximum point $y(t_1) = h$, $\upsilon(t_1) = 0$. Then $h = \upsilon_0 t_1 - \frac{gt_1^2}{2}$, $0 = \upsilon_0 - gt_1$, where t_1 is the rise time. Hence $\upsilon_0 = gt_1$, $\upsilon_0 = 9{,}81 \cdot 1{,}5 = 14{,}7\, m/s$.

The height to which the body rose is:

$$h = \upsilon_0 t_1 - \frac{gt_1^2}{2} = 14{,}7 \cdot 1{,}5 - \frac{9{,}8 \cdot 1{,}5^2}{2} = 11m$$

Answer: $\upsilon_0 = 14{,}7\, m/s$, $h = 11m$

2. During the time $t = 1$ min he train reduces its speed from $\upsilon_1 = 40km/h$ to $\upsilon_2 = 28km/h$. The movement of the train is assumed to be uniformly decelerated. Find the acceleration a of the train and the distance S, it travels during braking.

Known quantities:

$t = 1\min = 60\ s$

$\upsilon_1 = 40\,km/h = 11{,}1\,m/s$

$\upsilon_2 = 28km/h = 7{,}78m/s$

$a - ?,\ S - ?$

Problem solution

For uniformly decelerated motion:

$$\upsilon = \upsilon_0 - at,\ S = \upsilon_0 t - \frac{at^2}{2}.$$

According to the conditions of the problem, the following equation can be written: $\upsilon_2 = \upsilon_1 - at$. Then the acceleration is equal to:

$$a = \frac{\upsilon_1 - \upsilon_2}{t} = 0{,}055 m/s^2.$$

The distance travelled by the train during braking can be found using the formula:

$$S = \upsilon_1 t - \frac{at^2}{2} = 567 m.$$

Answer: $a = 0{,}055 m/s^2$, $S = 567 m$.

3. The ball was thrown vertically upwards with the initial speed $30 m/s$. Determine the time after which it will be at a height of $25 m$?

Known quantities:	Problem solution
$\upsilon_0 = 30 m/s$ $h = 25 m$ t -?	The upward movement of the ball is determined by the equation:

$$h = \upsilon_0 t - \frac{gt^2}{2} \Rightarrow \frac{gt^2}{2} - \upsilon_0 t + h = 0$$

Solving the quadratic equation and substituting numerical values, we get:

$$t_{1,2} = \frac{\upsilon_0 \pm \sqrt{\upsilon_0^2 - 2gh}}{g},$$

$$t_{1,2} = \frac{30 \pm \sqrt{30^2 - 2 \cdot 10 \cdot 25}}{10},\ t_1 = 5 s,\ t_2 = 1 s.$$

The body has been at this height twice: once when rising and once when falling.

Answer: $t_1 = 5 s$, $t_2 = 1 s$

1.3. Motion of a body thrown horizontally. Motion of a body thrown at an angle to the horizon

The motion of a body thrown horizontally is shown in Fig. 4. The movement of a body thrown at an angle to the horizon is shown in Fig. 5.

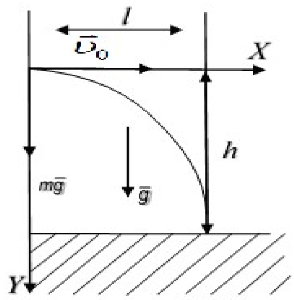


Figure 4

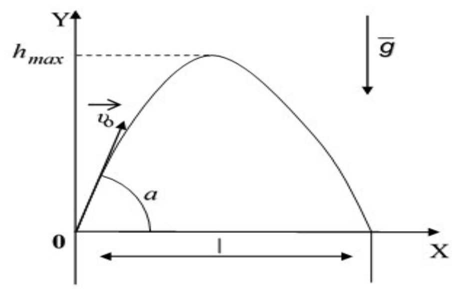


Figure 5

If there is no air resistance, then the movement of a body thrown at an angle to the horizon can be considered as the result of the addition of two simple movements:

uniform - along the horizontal axis OX, since there are no forces. This movement is described by the equation

$$\upsilon_x = \upsilon_{0x},\ x = x_0 + \upsilon_{0x}t;$$

uniformly accelerated - along the vertical axis OY under the action of gravity with acceleration g. Such movement is described by the equation

$$\upsilon_y = \upsilon_0 + gt,\ \ y = y_0 + \upsilon_{0y}t + \frac{gt^2}{2}.$$

At the maximum height, the projection of the movement speed on the OY axis is zero:

$$\upsilon_y = 0 \Rightarrow \upsilon_{0y} + gt = 0.$$

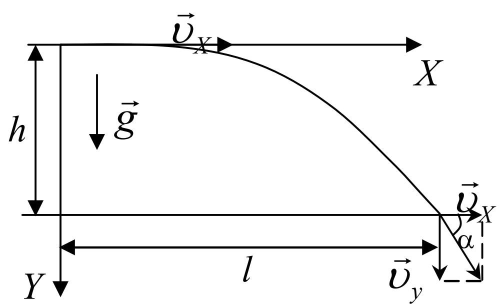
The modulus of the speed of movement at a given moment of time is determined by the Pythagorean Theorem: $\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2}$. The shape of the body's trajectory is a parabola.

Problem-solving examples

1. A stone thrown horizontally hits the ground after time $t = 1{,}5\,s$ at a distance of $8\ m$ meters horizontally from the place of throwing. Calculate the initial speed of the stone and the height from which it was thrown. Determine the speed with which it fell to the ground and the angle that is tangent to the trajectory of the stone with the horizon at the point of its fall to the ground. Neglect air resistance.

Known quantities:
$t = 1{,}5\, s;\, l = 8\ m$

$h, \upsilon_0, \upsilon, \varphi$ -?



Problem solution

The movement of the stone can be seen as the result of the addition of two simple movements. Motion along the vertical axis OY is uniformly accelerated with an acceleration of $\vec{g}$, therefore the equation of motion has the form: $y = y_0 + \upsilon_{0y} t + \dfrac{gt^2}{2}$.

The initial and boundary conditions have the form: $y = h\ \ y_0 = 0$, $\upsilon_{0y} = 0$.

Therefore, the movement of stones vertically is described by the equation

$$y = h = \frac{gt^2}{2} = \frac{9{,}8 \cdot 1{,}5^2}{2} = 11\ m.$$

The movement of stones horizontally is uniform, therefore: $x = x_0 + \upsilon_{0x} t$.

The initial and boundary conditions have the form $x = l,\ x_0 = 0,\ \upsilon_{0x} = \upsilon_0$,

$$x = l = \upsilon_0 t \Rightarrow \upsilon_0 = \frac{l}{t} = \frac{8}{1{,}5} = 5{,}3\ m/s.$$

Since $\upsilon_{0y} = 0$, then $\upsilon_y = gt$.

The speed of stones when falling to the ground

$$\upsilon = \sqrt{\upsilon_x^{\ 2} + \upsilon_y^{\ 2}} = \sqrt{\upsilon_x^{\ 2} + g^2 t^2} = \sqrt{5,3^2 + 9,8^2 1,5^2} = 15,6\ m/s.$$

The angle φ is the angle between the velocity vector and the vector of its horizontal component $\vec{\upsilon}_x$. It can be seen from the figure that

$$\cos\varphi = \frac{\upsilon_x}{\upsilon} = \frac{5,3}{15,6} = 0,34. \text{ Then } \varphi = 70^0.$$

Answer: $h = 11\ m$; $\upsilon_x = \upsilon_0 = 5,3\ m/s$; $\upsilon = 15,6\ m/s$; $\varphi = 70^0$.

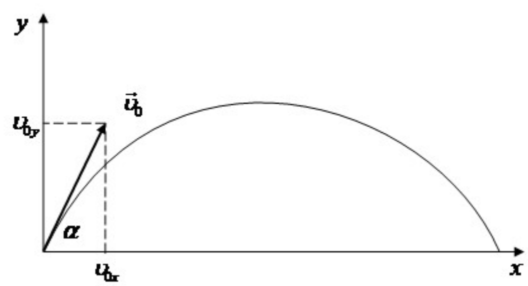
2. The ball was thrown at a speed of $\upsilon_0 = 12\ m/s$ at an angle of $\alpha = 35^0$ to the horizon. Determine how high the ball will rise; at what distance from the throwing place will the ball fall to the ground; how long it will be in motion. Ignore air resistance.

Known quantities:

$\upsilon_0 = 12\ m/s$

$\alpha = 35^0$

$h - ?\ S - ?\ t - ?$



Problem solution

Let's decompose the initial velocity υ_0 of the body into two components: $\upsilon_{0x} = \upsilon_0 \cos\alpha$, $\upsilon_{0y} = \upsilon_0 \sin\alpha$. In this case, the movement of the body can be considered as the sum of two independent movements (horizontal and vertical). In the horizontal direction, no forces act on the body (air resistance can be neglected). This movement is uniform and rectilinear. The horizontal movement of the ball is described by the equation: $S_x = (\upsilon_0 \cos\alpha)t$.

Vertical motion is free fall. It is described by the equation:

$$S_y = (\upsilon_0 \sin\alpha)t - \frac{gt^2}{2}.$$

The vertical component of velocity is: $\upsilon_y = \upsilon_0 \sin\alpha - gt$.

The body will move to the upper point of the trajectory until its vertical component of velocity υ_y becomes zero: $\upsilon_0 \sin\alpha = gt_1$, where t_1 is the time of movement to the upper point. From here, the time of movement to the upper point can be determined by the formula: $t_1 = \dfrac{\upsilon_0 \sin\alpha}{g}$.

The height to which the ball will rise is $h = \left(\upsilon_0 \sin\alpha\right)t_1 - \dfrac{g{t_1}^2}{2}$. Substituting the expression for t_1 , we get

$$h = \frac{{\upsilon_0}^2 \sin^2\alpha}{g} - \frac{g{\upsilon_0}^2 \sin^2\alpha}{2g^2} = \frac{{\upsilon_0}^2 \sin^2\alpha}{2g} = \frac{12^2\left(\sin 35\right)^2}{2\cdot 9{,}81} = 2{,}41\ m$$

The full flight time of the ball is equal: $t = \dfrac{2\upsilon_0 \sin\alpha}{g}$.

$$t = \frac{2\cdot 12\cdot \sin 35^0}{9{,}81} = 1{,}4\ s.$$

The distance S from the place of throwing to the point where the ball will fall to the ground can be determined by the equation

$$S = \upsilon_x t\ = \upsilon_0 \cos\alpha \frac{2\upsilon_0 \sin\alpha}{g} t = \frac{\upsilon_0^2\, 2\sin\alpha\cos\alpha}{g},$$

$$S = \frac{{\upsilon_0}^2\, 2\sin 35^0 \cos 35^0}{9{,}81} = 13{,}55\ m$$

<u>Answer:</u> $h = 2{,}41\ m$; $S = 13{,}55 m$; $t = 1{,}4 s$.

3. A body is thrown from the surface of the earth at an angle of 30^0 to the horizon with a speed equal to $20 m/s$. Determine the module of movement of the body from the beginning of the movement to the highest point of the trajectory. Acceleration of free fall is equal $g = 9{,}8\, m/s^2$.

Known quantities	Problem solution
$\alpha = 30^0$ $\upsilon = 20\, m/s$ $s - ?$	

Problem solution

The body movement module from the beginning of the movement to the highest point of the trajectory can be found by the Pythagorean Theorem: $s = \sqrt{s_x^2 + s_y^2}$. Along the OX axis, the movement is uniform: $\upsilon_x = \upsilon_{0x}$, $s_x = \upsilon_{0x} t$.

$$\upsilon_{0x} = \upsilon_0 \cos\alpha = 20 \cdot \frac{\sqrt{3}}{2} = 10\sqrt{3},\ s_x = \upsilon_{0x} t = 10\sqrt{3} \cdot t.$$

Along the OY axis, the motion is uniformly accelerated with acceleration $\vec{g}$:

$$\upsilon_y = \upsilon_{0y} - gt,\ s_y = \upsilon_{0y} t - \frac{gt^2}{2},\ \Rightarrow\ \upsilon_{0y} = \upsilon_0 \sin\alpha = 20 \cdot \frac{1}{2} = 10\, m/s;$$

$$\upsilon_y = 10 - 10t;\ s_y = 10t - \frac{10t^2}{2} = 10t - 5t^2.$$

At the highest point of the trajectory, the vertical component of the velocity is $\upsilon_y = 0$. From here we will find the time of movement to the highest point of the trajectory: $0 = 10 - 10t \Rightarrow t = 1s$. Substituting this value for time, we get:

$$s_x = \upsilon_{0x} t = 10\sqrt{3} \cdot t = 10\sqrt{3} m;\ s_y = 10t - 5t^2 = 5m \Rightarrow$$

$$s = \sqrt{s_x^2 + s_y^2} = \sqrt{300 + 25} = 18\ m.$$

<u>Answer</u>: $s = 18\, m$

1.4. Uniform movement in a circle

Uniform motion in a circle is curvilinear motion, in which the trajectory of the body's motion is a circle and for any equal time intervals, the body passes the same paths.

The **period** T **of rotation** is equal to the time interval during which a body moving uniformly in a circle makes one complete turn:

$$T = \frac{t}{N},$$

where t is the observation time, N is the number of complete turns made during this time. The SI unit of the rotation period is a second, s. $[T] = s$

Rotational frequency n – is a scalar value that is equal to the number of turns per unit of time: $n = \frac{N}{t}$.

The SI unit of rotational frequency is a turn per second (1/s): $[n] = \frac{1}{s} = Hz$

Relationship between frequency and period of rotation: $n = \frac{1}{T}$.

Instantaneous velocity $\vec{\upsilon}$ is the velocity of movement at a given point. This velocity is directed tangentially to the circle and perpendicular to the radius of the circle. The direction of the instantaneous velocity changes all the time, and its modulus remains unchanged. The module of movement velocity is determined by the formula: $\upsilon = \frac{l}{t}$, where l – is the path traveled by the body (arc length), t is the time for which this path was completed.

Linear velocity υ – is a scalar value, the value of which is equal to the modulus of the instantaneous velocity. For a time equal to one period $t = T$, the body moves a path equal to the length of the circle ($l = 2\pi R$, where R is the radius of the circle), so the linear velocity can be calculated using the formulas:

$$\upsilon = \frac{2\pi R}{T},\ \upsilon = 2\pi R n.$$

Angular velocity ω is a physical quantity that is numerically equal to the angle of rotation of the radius per unit of time:

$$\omega = \frac{\varphi}{t}.$$

The SI unit of angular velocity is radian per second (rad/s, or s^{-1}): $[\omega] = \frac{rad}{s} = \frac{1}{s}$

For a time equal to one period: $t = T$, the radius returns by 360^0 ($\varphi = 2\pi$), so the angular velocity can be calculated by the formula:

$$\omega = \frac{2\pi}{T} = 2\pi n.$$

Angular and linear velocities are related by a ratio: $\upsilon = \omega R$.

Centripetal acceleration $\vec{a}_n$ is directed towards the center of the circle. Centripetal acceleration is calculated by the formula:

$$a_n = \frac{\upsilon^2}{R} = \omega^2 R.$$

Problem-solving examples

1. A wheel with a diameter of $30\,cm$ makes 360 turns in 3 minutes. Calculate the rotation period, frequency, angular and linear speed of the wheel rim.

Known quantities:

$d = 30\,cm = 0,3\,m$

$N = 360$

$t = 3\min = 180\,s$

$T - ?, n - ?, \omega - ?, \upsilon - ?$

Problem solution

The rotation period is: $T = \frac{t}{N}$,

$$T = \frac{180}{360} = 0{,}5\,s,$$

the frequency is equal to: $n = \frac{1}{T}$, $n = \frac{1}{0{,}5} = 2\,s^{-1}$,

the angular velocity is equal to $\omega = \frac{2\pi}{T} = 2\pi n$, $\omega = \frac{2 \cdot 3{,}14}{0{,}5} = 12{,}56\,rad/s$,

the linear speed is equal to $\upsilon = \frac{2\pi R}{T}$, $\upsilon = \frac{0{,}3 \cdot 3{,}14}{0{,}5} = 1{,}9\,m/s$.

Answer: $T = 0{,}5\,s$, $n = 2\,s^{-1}$, $\omega = 12{,}56\,rad/s$, $\upsilon = 1{,}9\,m/s$.

2. The rotation frequency of the airplane propeller is $1500\,\text{min}^{-1}$. Find the number of revolutions made by the screw on the path $90\,km$ at a speed of $180\,km/h$

Known quantities:	*Problem solution*
$n = 1500\,\text{min}^{-1} = 25\,s^{-1}$ $\upsilon = 180\,km/h = 50\,m/s$ $l = 90\,km = 90 \cdot 10^3\,m$ $N - ?$	The flight time of the plane is determined by the formula: $t = \frac{l}{\upsilon}$.

The number of turns made by an airplane propeller:

$$N = tn = \frac{l}{\upsilon} n \Rightarrow N = \frac{90 \cdot 10^3}{50} 25 = 45000 .$$

Answer: $N = 45000$ turns

3. The time of one rotation of the Earth around the axis is equal to $24\,h$. Calculate the angular and linear speed of rotation at the equator. Take the radius of the Earth equal to $6400\,км$.

Known quantities:	*Problem solution*
$T = 24h = 24 \cdot 3600 = 86400s$ $R = 6400km = 6,4 \cdot 10^6 m$ $\omega - ?\ \upsilon - ?$	We consider the Earth's rotation to be uniform.

The angular velocity of rotation of the Earth around its axis can be calculated using the formula: $\omega = \frac{2\pi}{T}$. By substituting numerical data, we get:

$$\omega = \frac{2 \cdot 3,14}{86400} = 7 \cdot 10^{-5} rad / s.$$

The linear speed of rotation can be found from the formula for the relationship between the linear speed and the angular speed: $\upsilon = \omega R$.

$$\upsilon = 7 \cdot 10^{-5} \cdot 6,4 \cdot 10^6 = 448 m / s.$$

Answer: $\omega = 7 \cdot 10^{-5} rad / s$; $\upsilon = 448 m / s.$

1.5. Problems and tests for self-solving on the topic "Kinematics"

Problem Solving Guidelines. When solving problems in which the kinematic movement of bodies is considered, students should choose a frame of reference, write down the kinematic equations of motion in the general case in vector form and project these equations onto the coordinate axes. For problems in which kinematic motion in the field of gravity is considered, it is convenient to choose a coordinate system so that part of the projections of the velocity or acceleration vector on them is equal to zero. This makes it easier to solve problems.

If the body simultaneously participates in several movements, then individual movements are considered as if they occur independently of each other. At the same time, the movement, speed and acceleration of bodies are a vector sum of individual types of movement. For example, the movement of bodies thrown at an angle to the horizon is considered as the result of the superimposition of two simultaneous rectilinear movements along axes directed along the Earth's surface and normal to it. Taking this into account, solving problems of this type begins with finding the projections of the initial velocity vector along these axes and then constructing an equation for each direction. A body thrown at an angle to the horizon, in the absence of air resistance and a small initial speed, flies along a parabola, and the time of movement along the OX axis is equal to the time of movement along the OY axis, since both of these movements occur simultaneously. In a complex system of kinematic equations, the number of unknowns must be equal to the number of equations. When moving bodies in a circle, it is convenient to direct one of the coordinate axes in the direction of normal (centripetal) acceleration, that is, to the center of the circle.

1. The body moves with uniform acceleration along the OX axis. At the initial moment, the body is at the origin of coordinates, the projection of the initial speed of movement is $\upsilon_{0x} = 4m/s$, and the projection of acceleration is $a_x = -2m/s^2$. Determine the speed of the body after a time equal to 4 s after the start of movement.

2. A body freely falls vertically downwards from a height of $20m$ with no initial velocity. Determine the time of the fall and the path that the body passed during the last second of the fall.

3. The motion of two bodies is given by the equations: $x_1 = 15 + t^2$ i $x_2 = 8t$. Build a dependency graph $x(t)$. Calculate the time after which the meeting will take place and the coordinate of this point.

4. A ball is thrown from the surface of the earth at an angle of 40^0 to the horizon with a speed of $25m/s$. Calculate the height to which the ball will raise; the distance from the throwing place at which the ball will fall to the ground and the time of the ball's movement. Ignore air resistance.

5. The rectilinear motion of a material point is described by the equation $x = 0,5t - 8t^2$. Calculate the time when the velocity is zero.

6. The equations of motion of two material points along a straight line have the form: $x_1 = 12t - 4t^2$. Calculate the time when the velocities of these points will be the same.

7. The equation of motion of a material point along a straight line has the form $x = 4 + 2t - 5t^2$. Calculate the time when the velocity of the point will be zero.

8. The point moved during time $15s$ at a speed of $15m/s$, then during time $15s$ at a speed of $25m/s$. What is the average speed of the point?

9. The equation of rectilinear motion has the form $x = 2t + 6t^2$. Plot graphs of the dependence of the coordinate on time for this movement.

10. A stone falls from a height of $1,25km$. What path S will the stone pass in the last second of its fall?

11. A body freely falls vertically downwards from a height of $20m$ without initial velocity. Determine the time of fall.

12. The body moves with an acceleration of $2m/s^2$. The initial speed of the body is $18\ km/h$. Determine the time when the speed of the body will be equal to $25m/s$.

13. The body moves with uniform acceleration and covers a distance of $3,9m$ in the tenth second. Find the acceleration of the body if the initial velocity is $2m/s$.

14. Determine the initial speed of a body thrown vertically upwards, if it passes the height mark $60m$ twice with a time interval of $4s$. Neglect air resistance.

15. A body thrown vertically downwards with an initial velocity of $19,6m/s$. In the last second, the body passed a fourth part of the entire path. Determine the time of the fall of the body and its final speed.

Tests

1. What is the motion of a body called if the acceleration is $-3m/s^2$?

A	B	C	D
uniform	uniformly accelerated	uniform slow	motionless

2. Approaching the station, the train decelerated by $10m/s$ in time $20s$. With what acceleration was the train moving?

A	B	C	D
$-0{,}5m/s^2$	$2m/s^2$	$0{,}5m/s^2$	$-2m/s^2$

3. What is the motion of a body if it passes a distance of $3{,}5m$ in every second?

A	B	C	D
uniform	uniformly accelerated	uniform slow	motionless

4. A body moves without initial velocity with acceleration $0{,}4m/s^2$. Determine the displacement of the body after time $5s$ after the start of the movement.

A	B	C	D
$20m$	$10m$	$5m$	$2m$

5. The motion of the body is described by the equation $x = 4 - 3t + 4t^2$, where all values are expressed in SI units. What is the modulus of acceleration of the body?

A	B	C	D
$2m/s^2$	$4m/s^2$	$6m/s^2$	$8m/s^2$

6. The motion of the body is described by the equation $x = 4 - 3t + 4t^2$, where all values are expressed in SI units. Determine the projection of the velocity of the body on the OX axis $2s$ after the start of the movement.

A	B	C	D
$-6 m/s$	$5 m/s$	$6 m/s$	$8 m/s$

7. Determine the type of movement of the body if its speed increases by the same amount every second.

A	B	C	D
uniform	uniformly accelerated	uniform slow	motionless

8. The body moves in a circle with a radius of $5 m$, the period of its rotation is $10 s$. Determine the speed of the body.

A	B	C	D
$2 m/s$	$\pi m/s$	$4 m/s$	$4\pi\ m/s$

9. The change in the speed of a body moving in a straight line along the OX axis is described by the equation $\upsilon = 7 - 2t$, where all values are expressed in SI units. Calculate the acceleration of the body.

A	B	C	D
$-2 m/s^2$	$2 m/s^2$	$-4 m/s^2$	$7 m/s^2$

10. A ball is thrown vertically upwards with a speed of $10 m/s$. Determine how long it will take the ball to return to its starting point? Acceleration of free fall should be taken into account equal to $10 m/s^2$.

A	B	C	D
$1s$	$2s$	$4s$	$10s$

11. A third of the time the body moved at a speed of $54 km/h$, the rest of the time at a speed of $45 m/s$. Calculate the average speed of the body for the entire time of movement.

A	B	C	D
$35 m/s$	$30 m/s$	$40 m/s$	$20 m/s$

12. After a time of $10s$ after the start of uniformly accelerated motion, the velocity of the body is $0,5 m/s$. After what time from the start of the movement will the body acquire a speed $2 m/s$?

A	B	C	D
$20s$	$30s$	$40s$	$50s$

13. The body was thrown with a speed of $20 m/s$ and at an angle of 60^0 to the horizon. Calculate the modulus of velocity and acceleration of the body at the upper point of the trajectory.

A	B	C	D
$10 m/s; 0 m/s^2$	$10 m/s; 10 m/s^2$	$0 m/s; 0 м/s^2$	$0 m/s; 10 m/s^2$

14. Two bodies were thrown at the same angle to the horizon. The flight range of the first body is 4 times greater than the flight range of the second. Determine how many times the initial velocities of the bodies differ?

A	B	C	D
$\frac{\upsilon_{01}}{\upsilon_{02}} = 4$	$\frac{\upsilon_{01}}{\upsilon_{02}} = 2$	$\frac{\upsilon_{01}}{\upsilon_{02}} = \sqrt{2}$	$\frac{\upsilon_{01}}{\upsilon_{02}} = \sqrt{8}$

15. Two bodies are thrown at angles of $\alpha_1 = 30^0$ and $\alpha_2 = 60^0$ to the horizon with the same initial velocities. Calculate how many times the maximum height of the second body is different from the maximum height of the first body.

A	B	C	D
2	$\sqrt{2}$	3	$\sqrt{3}$

2. DYNAMICS

2.1. Newton's laws

Dynamics is a branch of mechanics that studies the causes of changes in the motion of bodies as a result of their interaction.

The **phenomenon of inertia** is the phenomenon of a body maintaining a state of rest or uniform rectilinear motion under the conditions that it is not acted upon by other bodies and fields or their actions are compensated.

Motion by inertia is the movement of a body at a constant speed, provided that it is not acted upon by other bodies and fields or their actions are compensated.

An **inertial frame of reference** is a frame of reference relative to which a body keeps its speed of motion constant, if it is not acted upon by other bodies and fields or their actions are compensated. Frames of reference that move uniformly in a straight line relative to an inertial frame of reference are also inertial.

The **principle of relativity in classical mechanics** (Galileo's principle of relativity): all inertial frames of reference are equal, that is, in all these systems, any mechanical phenomena occur in the same way.

Inertia is a property of the body, which is that it takes some time to change the speed of the body's movement.

Mass m is a physical quantity that is a measure of inertia and gravity of a body. The SI unit of mass is kilogram, kg: $[m] = kg$

Force $\vec{F}$ is a vector quantity that is a measure of action on the body of other bodies, as a result of which the body acquires acceleration or changes its shape and size.

The SI unit of force is the Newton, N: $[F] = N$

Force is characterized by a numerical value, a direction in space and a point of application.

The **resultant force** $\vec{F}$ is a force that is equal to the vector sum of all forces acting on a given body: $\vec{F} = \vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n$.

Body momentum $\vec{p}$ is a vector value equal to the product of the body's mass and the speed of its movement: $\vec{p} = m\vec{\upsilon}$. The SI unit of body momentum is: $[p] = kg \cdot m/s$. The direction of the momentum of the body coincides with the direction of the speed of its movement.

Force impulse $\vec{F}t$ is a vector value equal to the product of the force times the action.

Newton's first law: there are such reference systems relative to which a body maintains a state of rest or uniform rectilinear motion, if it is not acted upon by other bodies and fields or their actions are compensated.

Newton's second law: the acceleration that a body acquires due to the action of a force is directly proportional to this force and inversely proportional to the mass of the body:

$$\vec{a} = \frac{\vec{F}}{m}, \qquad \vec{F} = m\vec{a}.$$

If several forces act on the body at the same time, then Newton's second law has the form:

$$\vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n = m\vec{a}.$$

The direction of the body's acceleration always coincides with the direction of the net forces applied to the body: $a \uparrow\uparrow \vec{F}$.

If the resultant force is zero, then the body moves uniformly and in a straight line.

A body accelerates if the direction of the resultant force coincides with the direction of the body's motion. A body slows down if the direction of the resultant force is opposite to the direction of the body's motion.

Newton's second law in momentum form: the momentum of a force acting on a body is equal to the change in momentum of the body:

$$\vec{F}t = m\upsilon - m\upsilon_0, \qquad \vec{F}t = \Delta\vec{p}.$$

Newton's third law: forces with which bodies act on each other are directed along the same straight line, equal in magnitude and opposite in direction:

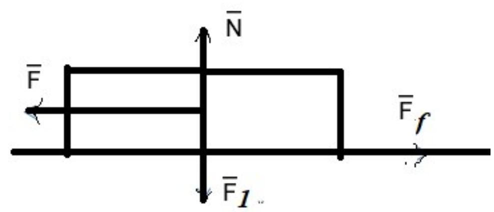
$$\vec{F}_1 = -\vec{F}_2, \qquad F_1 = F_2.$$

Forces of interaction of bodies arise in pairs and have the same nature. Interaction forces are applied to different bodies and therefore cannot be represented as a resultant force.

Problem-solving examples

1. A car with a mass of $4m$ starts from a standstill and covers the first 200 m in $10s$. Find the thrust of the engine if the coefficient of friction is $\mu = 0{,}1$.

Known quantities:
$m = 4m = 4000\,kg$
$\mu = 0{,}1$
$t = 10\,s$
$S = 200\,m$
$F - ?$



Problem solution

The following forces are applied to the car: gravitational force $m\vec{g}$, support reaction force $\vec{N}$, friction force $\vec{F}_f = \mu\vec{N}$ $\vec{F} = m\vec{a}$ and engine thrust $\vec{F}$.

Under the action of these forces, the body moves with acceleration $\vec{a}$. Newton's second law in vector form has the form: $\vec{F} + \vec{F}_f + \vec{N} + m\vec{g} = m\vec{a}, \quad m\vec{g} = \vec{F}_1.$

Let's project this equation onto the coordinate axes:

$OX: F - F_f = ma$. Then the engine thrust is $F = ma + F_f$

$OY: N - mg = 0$. Whence $N = mg$

The friction force is: $F_f = \mu N = \mu mg$. The engine thrust is: $F = m(a + \mu g)$.

Let's find the acceleration of the car under the condition that the initial speed is zero:

$S = \dfrac{at^2}{2} \Rightarrow a = \dfrac{2S}{t^2}$. $F = m(\dfrac{2S}{t^2} + \mu g)$

$$F = m(\frac{2S}{t^2} + \mu g) = 4000(\frac{2 \cdot 200}{10^2} + 0{,}1 \cdot 9{,}8) = 19920\,N$$

<u>Answer:</u> $F = 1{,}992 \cdot 10^4\,N$

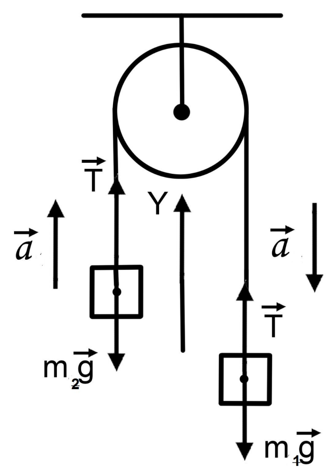
2. Weights with masses $m_1 = 2\,kg$ and $m_2 = 1\,kg$ are connected by a thread and thrown over a weightless block. Find the acceleration a, with which the weights move and the tension T of the string. Ignore friction in the block.

Known quantities:

$m_1 = 2\,kg$

$m_2 = 1\,kg$

$a - ?, T - ?$



Problem solution

Suppose that the thread is weightless and inextensible, then the force of tension on the thread is the same at all its points. The acceleration of the movement of weights is also the same, because due to the inextensibility of the thread, in the same time, the weights pass one way, i.e.:

$$S_1 = \frac{a_1 t^2}{2}, \quad S_2 = \frac{a_2 t^2}{2} \quad \Rightarrow \quad S_1 = S_2$$

Hence, $a_1 = a_2$. Let's write Newton's second law for the first and second weights in projections on the direction of motion:

$$m_1 g - T = m_1 a, \qquad (1)$$

$$T - m_2 g = m_2 a, \qquad (2)$$

$$a(m_1 + m_2) = g(m_1 - m_2)\ , \quad a = \frac{g(m_1 - m_2)}{(m_1 + m_2)}, \qquad (3)$$

$$a = 3{,}27 m/s^2.$$

Substitute (3) into (1) $T = m_1 g(1 - \frac{m_1 - m_2}{m_1 + m_2}) = \frac{2gm_1m_2}{m_1 + m_2}$.

By substituting numerical values, we get $T = 13{,}06 N$

<u>Answer:</u> $T = 13{,}06 N$, $a = 3{,}27 m/s^2$.

3. Determine the force F that must be applied to the wagon standing on the rails so that the wagon starts moving with uniform acceleration and travels a distance of $11m$ in time $30s$? Mass of the wagon is 16 ton. During movement, the car is subjected to a frictional force F_f, equal to 0,05 of the force of gravity acting on it.

<u>Known quantities:</u>
$t = 30s$,
$S = 11m$
$m = 16 ton$
$F_f = 0{,}05mg$
$F - ?$

<u>Problem solution</u>

Two forces act on the body: the force of friction and the force applied to wagon. According to Newton's second law:

$$\vec{F} + \vec{F}_f = m\vec{a}$$

or in the projection on the axis x: $F - F_f = ma$, from here $F = F_f + ma$.

The force of friction is equal to $F_f = \mu mg$. Since the motion is uniformly accelerated and $\upsilon_0 = 0$, then the path is $s = \frac{at^2}{2}$, hence $a = \frac{2s}{t^2}$. According to the condition of the problem $F_f = 0{,}05mg$:

$$F = m\frac{2s}{t^2} + 0{,}05mg = 7{,}9 \cdot 10^3 N.$$

<u>Answer:</u> $F = 7{,}9 \cdot 10^3 N$

2.2. Gravitational forces

Gravity is a phenomenon that causes the attraction of all bodies in the universe.

The **forces of universal gravitation** are a measure of the gravitational interaction of bodies.

The **law of universal gravitation**: two material points are attracted to each other with forces whose modulus is directly proportional to the product of their masses (m_1, m_2) and inversely proportional to the square of the distance (r) between them:

$$F = G\frac{m_1 m_2}{r^2},$$

where $G = 6,67 \cdot 10^{-11} \dfrac{N \cdot m^2}{kg^2}$ is the gravitational constant. The force of gravity is directed towards the center of the Earth. Gravitational attraction forces act along a straight line connecting the centers of gravity of interacting bodies. These forces are equal in magnitude and opposite in direction.

Gravitational force is the force with which the Earth attracts a body of mass m.

The modulus of gravity can be determined in two ways:

1) According to the law of universal gravitation: $F = G\dfrac{m\ M}{r^2}$, where M is the mass of the Earth, $r = R + h$ is the distance from the body to the center of the Earth, R is the radius of the earth.

2) According to Newton's second law: $F = mg_h$, where g_h is the acceleration of free fall at height h.

Near the Earth's surface $h = 0$: $F = G\frac{m\,M}{R^2} = mg$, where $g = G\frac{M}{R^2} = 9{,}8\,m/s^2$ is the acceleration of free fall near the Earth's surface.

The **weight of the body** is the force with which the body, as a result of its attraction to the Earth, acts on the support or suspension. If the support is horizontal and is at rest or moves uniformly in a straight line, then the weight of the body coincides in magnitude and direction with the force of gravity: $\vec{P} = m\vec{g}$. The weight is applied to the support or suspension, and the force of gravity is applied to the center of mass of the body.

If the acceleration of the body is directed vertically upwards, then its weight increases: $P = m(g + a)$.

If the acceleration of the body is directed vertically downward, then its weight decreases: $P = m(g - a)$.

If the acceleration of the body is equal to the acceleration of free fall, then the weight of the body is zero (state of weightlessness).

Weightlessness is a state of the body in which there is no internal tension due to the force of gravity.

Problem-solving examples

1. An artificial satellite moves around the Earth in a circular orbit. Find the speed of the satellite if the satellite moves at a height equal to the radius of the Earth.

Known quantities	*Problem solution*
$h = R$	
$\upsilon - ?$	

The satellite moves in a circular orbit due to the action of gravity

$$F = G\frac{m\,M}{(R+h)^2},$$

where M is the mass of the Earth, R is the radius of the Earth. This force gives the satellite centripetal acceleration: $a_n = \dfrac{\upsilon^2}{(R+h)^2}$.

According to Newton's second law: $F = ma_n$.

$$G\frac{m\,M}{(R+h)^2} = \frac{m\upsilon^2}{(R+h)} \Rightarrow \upsilon = \sqrt{G\frac{M}{(R+h)}}.$$

Since $g = G\dfrac{M}{R^2}$, and from the condition of the problem $h = R$, then:

$$\upsilon = \sqrt{\frac{gR^2}{2R}} = \sqrt{\frac{gR}{2}}.$$

<u>Answer:</u> $\upsilon = \sqrt{\dfrac{gR}{2}}$

2. At what distance from the Earth's surface will the gravitational force of the spaceship become 100 times less than on the Earth's surface?

<u>Known quantities:</u>

$\dfrac{F_1}{F_2} = 100$

$x - ?$

<u>Problem solution</u>

We denote the radius of the Earth as R_E and the distance of the ship from the Earth's surface as x.

According to the law of universal gravitation let's write for the force of gravity on the Earth's surface: $F_1 = G\frac{m_s M_E}{{R_E}^2}$, where M_E is the mass of the Earth, m_s is the mass of the ship, $G = 6{,}67 \cdot 10^{-11}\,\frac{N \cdot m^2}{kg^2}$ is the gravitational constant.

The force of gravity at a distance of x of the ship from the surface of the Earth: $F_2 = G\frac{m_s M_E}{(R_E + x)^2}$. From the condition of the problem, we get: $\frac{F_1}{F_2} = \frac{(R_E + x)^2}{R_E^2} = 100$.

Hence $(R_E + x)^2 = 100 R_E^2$. Solving the quadratic equation with respect to x, we get that the distance is equal to: $x = 9R_E$.

Answer: $x = 9R_E$.

3. With what acceleration should the weight be lifted so that its weight doubles? With what acceleration should the weight be lowered so that its acceleration is reduced by two?

Known quantities:

$P_2 = 2P_1$

$a_1, a_2 - ?$

Problem solution

Let's write Newton's second law in the projection on the vertical axis (the positive direction of the axis considered directed upwards):

$$ma = N - mg.$$

According to Newton's third law, the weight of the weight P is equal to the reaction force of the support N:

$$P = N = m(g + a).$$

When the weight is doubled: $P = 2mg = m(g + a_1)$, $a_1 = +g$.

When the weight is reduced by half: $P = \frac{mg}{2} = m(g + a_2)$. Hence $a_2 = -\frac{g}{2}$.

Answer: $a_1 = +g$, $a_2 = -\frac{g}{2}$.

2.3. Force of friction

The **force of friction** is a force that occurs during the actual or potential movement of one body on the surface of another and prevents this movement. A distinction is made between the force of sliding friction and the force of rest friction, the force of rolling friction, and viscous friction (the resistance force of the medium).

The **force of sliding friction** $\vec{F}_f$ occurs when one body slides over the surface of another. The direction of the sliding friction force is opposite to the speed of the body's movement. The modulus of the sliding friction force is directly proportional to the strength of the normal reaction of the support N:

$$F_f = \mu N,$$

where μ is the coefficient of sliding friction, μ is a dimensionless quantity.

The **coefficient of sliding friction** depends on the substance of the bodies that are compressed; states of surfaces that compress bodies; speed of sliding of one body relative to another.

The **force of rest friction (**static friction) is the same in magnitude and opposite in direction to the external force F, that tries to bring the body out of rest: $F_{fS} = F$.

The maximum value of the modulus of friction at rest is: $F_{fS\max} = \mu N$.

Viscous friction occurs when a solid moves in a liquid or gas. The force of viscous friction is directed against the speed of movement of the body. The modulus of the force of viscous friction depends on the speed of movement of the body:

at low speeds: $F_r = \beta_1 \upsilon$,

at high speeds: $F_r = \beta_2 \upsilon^2$,

where β_1, β_2 are the coefficients of proportionality between the modulus of the viscous friction force and the speed of body movement, they characterize the flow around the surface of the body and the viscosity of the medium.

Problem-solving examples

1. Loads with masses $m_1 = 2\,kg$ and $m_2 = 3\,kg$ are connected by a thread, which is thrown over a fixed block fixed at the end of the table. Coefficient of friction between the load and the horizontal surface is $\mu = 0,2$. Ignore the mass of the block and thread. Determine the acceleration of loads and the force N with which the block acts on the axis.

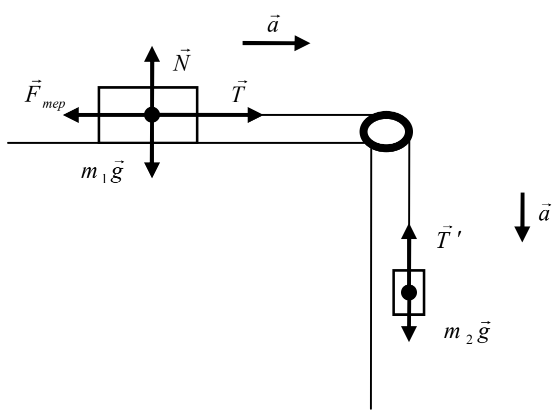
<u>Known quantities:</u>

$m_1 = 2\,kg$

$m_2 = 3\,kg$

$\mu = 0,2$

$a - ?,\ N - ?$



Problem solution

The figure shows the forces acting on the bodies and on the axis of the block and the direction of acceleration. We write down Newton's second law for both bodies:

for the first body: $\vec{a} = \dfrac{\vec{T} + m_1\vec{g} + \vec{N} + \vec{F}_f}{m_1}$

In the projection to the direction of acceleration:

$am_1 = T - F_f$, $F_f = \mu N = \mu m_1 g$;

$am_1 = T - \mu m_1 g$.

for the second body: $\vec{a} = \dfrac{m_2\vec{g} + \vec{T}'}{m_2}$ and in the projection to the direction of acceleration:

$$am_2 = m_2 g - T'.$$

Let's take into account that according to Newton's third law $\left|\vec{T}\right| = \left|\vec{T}'\right|$. Then

$$am_1 = T - \mu m_1 g,$$

$$am_2 = m_2 g - T.$$

We solve the system with respect to the acceleration value:

$$a = \frac{m_2 g - \mu m_1 g}{m_1 + m_2} = \frac{30 - 0,2 \cdot 20}{5} = 5,2 \text{ m/s}^2.$$

The resulting expression for the acceleration could be written immediately by Newton's second law. Indeed, the resulting projection of external forces on the direction of acceleration (on the thread) is in the numerator, and the mass of the entire system is in the denominator.

The force N is equal to the vector sum of mutually perpendicular tension forces. According to the Pythagorean Theorem, we get

$$N=\sqrt{T^2+T^2}=T\sqrt{2}=m_2(g-a)\sqrt{2}=3\cdot 4{,}8\cdot\sqrt{2}=20{,}4\ \text{N}$$

Answer: $a=5{,}2\ \text{m/s}^2$, $N=20{,}4\ \text{N}$

2. The tram moves with an acceleration of $49\ cm/s^2$. Find the coefficient of friction, if it is known that 50% of the engine's power is used to overcome the forces of friction and 50% to increase the speed of movement.

Known quantities:

$a=49\cdot 10^{-2}\ m/s^2$

$0{,}5N=F_T$

μ-?

Problem solution

Engine power can be calculated using the formula: $N=F\upsilon$,

where F is an active force, υ is the speed of movement. According to the condition of the problem, half of the power goes to overcome the friction force, i.e. $\frac{N}{2}=\mu mg\upsilon$ (where μ is the coefficient of sliding friction), and the second half – on increasing the speed of movement, that is $\frac{N}{2}=ma\upsilon$.

Hence $\mu mg\upsilon=ma\upsilon$, so we get:

$$\mu=\frac{a}{g}=\frac{49\cdot 10^{-2}}{9{,}81}=0{,}05.$$

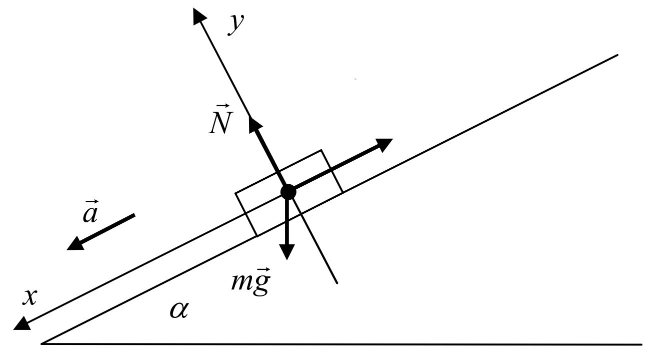
Answer: $\mu=0{,}05$.

3. A body slides along an inclined plane that makes an angle of 45^0 with the horizon. After passing a distance of $50cm$, the body acquires a speed of $3m/s$. Calculate the coefficient of friction of the body on the plane.

Known quantities:

$\alpha = 45^0$
$S = 50cm = 0,5m$
$\upsilon = 3m/s$

$\mu - ?$



Problem solution

A body sliding on an inclined plane is acted upon by the reaction force of the support $\vec{N}$, the force of friction $\vec{F}_f$ and the force of gravity $m\vec{g}$. The body moves with acceleration $\vec{a}$. Newton's second law has the form:

$$\vec{N} + \vec{F}_f + m\vec{g} = m\vec{a}.$$

Let's write Newton's second law in projections on the axis:

$$OX: -F_f + 0 + mg\sin\alpha = ma,$$

$$OY: 0 + N - mg\cos\alpha = 0.$$

Let's take into account that:

$$F_f = \mu N = \mu mg\cos\alpha,$$

$$-\mu mg\cos\alpha + mg\sin\alpha = ma \Rightarrow \mu = \frac{g\sin\alpha - a}{g\cos\alpha},$$

$$\upsilon_0 = 0,$$

$$S = \frac{\upsilon^2 - \upsilon_0^2}{2a} = \frac{\upsilon^2}{2a} \Rightarrow a = \frac{\upsilon^2}{2S}.$$

Let's substitute the acceleration a in the formula for μ:

$$\mu = \frac{g\sin\alpha - \frac{\upsilon^2}{2S}}{g\cos\alpha} = tg\alpha - \frac{\upsilon^2}{2Sg\cos\alpha} = 0,27.$$

Answer: $\mu = 0,27$.

2.4. Elastic forces. Hooke's law

Elastic forces are forces that arise when the body is deformed and prevent this deformation.

Deformation of a solid body is called a change in the shape and volume of the body under external influence. Elastic and plastic deformations are distinguished. The elastic deformation completely disappears after the termination of the action of external forces. Plastic deformation does not disappear after the termination of the action of external forces.

Hooke's law: the elastic force during elastic deformation is directly proportional to the absolute elongation and is directed opposite to it

$$F_{el} = -kx,$$

where k is rigidity, $k = \frac{SE}{l_0}$, S is the cross-sectional area of the body, l_0 is the initial length of the body, E is the modulus of elasticity or **Young's modulus**.

The unit of rigidity in SI is Newton per meter, $\frac{N}{m}$: $[k] = \frac{N}{m}$

The unit of Young's modulus in SI is Newton per square meter, $\frac{N}{m^2}$: $[E] = \frac{N}{m^2}$

Absolute elongation $x(\Delta l)$ is the difference between the final and initial lengths of the body:

$$x = l - l_0 = \Delta l.$$

Relative elongation ε of the body is the ratio of the absolute elongation to its initial length

$$\frac{|\Delta l|}{l_0} = |\varepsilon|.$$

Problem-solving examples

1. A load with a mass of $2\,kg$ was suspended from a spring with a stiffness of $500\,N/m$. Find the elongation of the spring. The load is considered stationary.

Known quantities:
$k = 500\,N/m$
$m = 2\,kg$
$x - ?$

Problem solution

The weight of a stationary load is equal to the force of gravity acting on the load/

The elongation of a spring can be found using Hooke's law: $F_{el} = kx$, where F_{el} is the modulus of elasticity, x is the modulus of elongation, k is the spring stiffness. From here we get the elongation of the spring: $x = \frac{F}{k} = \frac{2 \cdot 9{,}8}{500} = 0{,}04\,m$.

Answer: $x = 0{,}04\,m$

2. The stiffness of one spring is k_1, and the other is k_2. What is the stiffness of a spring composed of these two springs connected in series?

Known quantities:
$k_1,\ k_2$
$k - ?$

Problem solution

Since the spring is in equilibrium, the forces stretching the two springs in series are equal to each other.

According to Hooke's law, the elongation of the first spring is $\Delta l_1 = \frac{F}{k_1}$. The elongation of the second spring is equal to $\Delta l_2 = \frac{F}{k_2}$.

When connected in series, the total elongation of the spring can be determined by the formula $\Delta l = \Delta l_1 + \Delta l_2$.

$\Delta l = \frac{F}{k_1} + \frac{F}{k_2} = F(\frac{1}{k_1} + \frac{1}{k_2})$. Since $F = k\Delta l$, then $\Delta l = \frac{F}{k}$, where $k = \frac{k_1 k_2}{k_1 + k_2}$.

Answer: $k = \frac{k_1 k_2}{k_1 + k_2}$.

3. Calculate the elongation of the tow rope with a stiffness of $100\,kN/m$ when towing a car with a mass of $2\,tons$ and acceleration $0,5\,m/s^2$. Neglect the force of friction.

Known quantities:

$k = 100\,kN/m = 10^5\,N/m$

$m = 2\,tons = 2000\,kg$

$a = 0,5\,m/c^2$

$\Delta l - ?$

Problem solution

According to Newton's third law, the force that stretches the cable is equal to the force acting on the car.

Since there are no frictional forces, no other forces act on the car in the horizontal direction, therefore $ma = F$. Let's write Hooke's law for the cable: $F = k\Delta l$. The elongation of the tow rope can be found using the formula: $\Delta l = \frac{F}{k} = \frac{ma}{k}$.

$$\Delta l = \frac{2000 \cdot 0,5}{10^5} = 10^{-2}\,m = 1\,cm.$$

Answer: $\Delta l = 1\,cm$.

2.5. Problems and tests for self-solving on the topic «Dynamics»

Problem solving guidelines. When solving dynamics problems, special attention should be paid to the analysis of the forces acting on the body. It is necessary to determine the forces acting on each body and depict the corresponding force vectors in the figure. Write down the basic equation of motion on the basis of Newton's second law first in vector form, and then, choosing a convenient way (depending on the condition of the problem) of the coordinate system, in projections onto the coordinate axes. The positive direction of the axes is chosen so that it coincides with the direction of acceleration of the body. The resulting system of equations should be supplemented, if necessary, with kinematic equations and specific expressions of forces (for example, frictional or elastic forces).

When solving problems about the motion of a system of connected bodies, if the bodies are connected by a thread, the mass of which can be neglected, then the tension force is considered to be the same along the entire length of the thread. If the thread is thrown over the block, then the tension forces on different sides of the block are the same in magnitude, only in the case when the mass of the block and the force of friction arising during its rotation can be neglected.

1. The body moves from an inclined plane that forms an angle of 30^0 with the horizon. Having passed the path $0,72\,m$, the body acquires a speed of $2\,m/s$. What is the coefficient of friction of the body on the plane?

2. The body moves from an inclined plane that forms an angle of 30^0 with the horizon. The dependence of the path traveled by the body on time is given by the equation: $s = 2t^2$. Calculate the coefficient of friction of the body on the plane.

3. An inclined plane with a length of $25m$ forms an angle of 30^0 with the horizon plane. The body, moving with uniform acceleration, shifts from this plane in $2s$. Determine the coefficient of friction of the body on the plane.

4. The slope of the iceberg is directed at an angle of 30^0 to the horizon. Moving down the slope from bottom to top, the body at some point has a speed of $10m/s$. The coefficient of sliding friction is equal to $0,05$. What speed will this body have after it returns to its original position?

5. In a car moving horizontally and in a straight line with acceleration $2m/s^2$, a load of mass $200g$ is hanging on a cord. Calculate the tension of the cord and the angle of deviation of the cord from the vertical.

6. During the movement of a car with a mass of 1 ton a frictional force equal to $0,2$ of its gravity acts on it. What is the traction force of a car engine in uniform motion?

7. Weights with masses $m_1 = 100g$ and $m_2 = 110g$ are connected by a string and thrown over a weightless block. Find the acceleration a, with which the weights move and the tension T of the string. Ignore friction in the block.

8. The radius of the planet Mars is $0,53$ Earth radii, and the mass is $0,11$ Earth mass. Determine the acceleration of free fall on Mars.

9. A boy with a mass of $50kg$, sliding down a hill on a sled, drove along a horizontal road to a stop on route $20m$ in $10s$. Determine the force of friction and the coefficient of friction.

10. Find the stiffness of the spring, which has lengthened by $4cm$ under the action of a force of $2N$.

11. A body slides along an inclined plane located at an angle of 30^0 relative to the horizon. Having passed the distance of $0,6m$, the body acquired a speed of $2m/s$. Determine the coefficient of friction of the body relative to the plane.

12. A body slides along an inclined plane located at an angle of 45^0 relative to the horizon. The dependence of distance on time is given by the equation $S = Bt^2$, where $B = 1,73m/s^2$. Determine the coefficient of friction of the body relative to the plane.

13. Two weights of masses $3\ kg$ and $5\ kg$ are connected by a string thrown over a weightless block. Find the tension of the thread and the acceleration of the weights.

14. Two identical weights of equal masses $1\ kg$ each are connected by a string thrown over a weightless block fixed to the edge of a table so that one weight hangs and the other slides on the table with a coefficient of friction of $0,1$. Calculate the tension of the string and the acceleration of the weights.

15. An elevator weighing $30kN$ rises with an acceleration of $0,5m/s$. Determine the tension of the rope with which the cage is lifted. What will be the tension of the rope with a uniform movement of the cage upwards?

Tests

1. Two forces acting at right angles to each other act on the body at the same time. One force is equal to $3N$, the second force is equal to $4N$. What is the modulus of the resultant forces acting on the body?

A	B	C	D
$7N$	$5N$	$3,5N$	$1N$

2. What will be the nature of the movement of the body if a force of constant magnitude and direction acts on it?

A	B	C	D
uniform	uniformly accelerated	uniform slow	motionless

3. What physical quantity quantitatively characterizes the effect of one body on another?

A	B	C	D
velocity	force	acceleration	mass

4. A body with a mass of $200\,g$ moves with an acceleration of $0,4\,m/s^2$. What force causes this acceleration to the body?

A	B	C	D
$0,08\,N$	$80\,N$	$0,02\,N$	$20\,N$

5. The body is acted upon by forces of $120\,N$ in opposite directions. What is the movement of the body?

A	B	C	D
uniform	uniformly accelerated	uniform slow	motionless

6 Describe the movement of the body, as if the vector sum of forces, acting on it, would be equal to zero.

A	B	C	D
uniform	uniformly accelerated	uniform slow	motionless

7. The resultant of all forces acting on a moving body is zero. What is the trajectory of this body?

A	B	C	D
Straight line	Parabola	Hyperbole	Circle

8. A body with a mass of $0,5\,kg$ was suspended from a spring with a stiffness of $40\,N/m$. What is the force of elasticity? The acceleration of free fall is assumed to be equal to $10\,m/s^2$.

A	B	C	D
$5\,N$	$20\,N$	$200\,N$	$400\,N$

9. What is the weight of the boy at the beginning of the rise in the elevator, which moves with an acceleration of $2\,m/s^2$? The mass of the boy is $40\,kg$. The acceleration of free fall is assumed to be equal to $10\,m/s^2$.

A	B	C	D
$42\,N$	$400\,N$	$420\,N$	$480\,N$

10. The elevator moves down with an acceleration of $3\,m/s^2$. There is a girl weighing $30\,kg$ in it. What is the weight of the girl? The acceleration of free fall is assumed to be equal to $10\,m/s^2$.

A	B	C	D
$390\,N$	$300\,N$	$210\,N$	$90\,N$

11. The change in velocity of a body moving in a straight line along the axis is described by the equation $\upsilon = 3 + 12t$, where all values are expressed in SI units.

Determine the projection on the OX axis of the force acting on the body whose mass is $100\ g$.

A	B	C	D
$1,2N$	$0,2N$	$3N$	$4N$

12. A body of mass $3kg$ moves uniformly in a circle. The centripetal acceleration of the body is $2m/s^2$. Determine the resultant of all forces acting on this body.

A	B	C	D
$1,5N$	$3N$	$5N$	$6N$

13. A force of $2N$ stretches a spring by $1cm$. What force must be applied to stretch two identical springs connected in series by $1cm$?

A	B	C	D
$1N$	$2N$	$3N$	$4N$

14. A force of $2N$ stretches the spring by $1cm$. What force must be applied to stretch two identical springs connected in parallel by $1cm$?

A	B	C	D
$1N$	$2N$	$3N$	$4N$

15. The mass of the first body is $300kg$. The mass of the second body is $1200kg$. Compare the acceleration of the bodies if the traction force of the second body is twice as large as the traction force of the first body.

A	B	C	D
$\frac{a_1}{a_2}=2$	$\frac{a_1}{a_2}=\frac{1}{2}$	$\frac{a_1}{a_2}=4$	$\frac{a_1}{a_2}=\frac{1}{4}$

3. LAWS OF CONSERVATION IN MECHANICS

3.1. Momentum. Law of conservation of momentum

The momentum (impulse) $\vec{p}$ **of the body** is called a vector equal to the product of the mass of the body and its speed

$$\vec{p} = m\vec{\upsilon}.$$

The **law of conservation of momentum**: in a closed system of bodies, the geometric sum of the momentum of the bodies before the interaction is equal to the geometric sum of the momentum of the bodies after the interaction:

$$\vec{p}_1 + \vec{p}_2 + ... + \vec{p}_n = const.$$

When solving problems, the law of conservation of momentum is written in the following form:

$$m_1\vec{\upsilon}_{01} + m_2\vec{\upsilon}_{02} + ... + m_n\vec{\upsilon}_{0n} = m_1\vec{\upsilon}_1 + m_2\vec{\upsilon}_2 + ... + m_n\vec{\upsilon}_n,$$

where $m_1\vec{\upsilon}_{01}, m_2\vec{\upsilon}_{02}, ...m_n\vec{\upsilon}_{0n}$ are the impulses of bodies to interact, $m_1\vec{\upsilon}_1, m_2\vec{\upsilon}_2, ...m_n\vec{\upsilon}_n$ are the impulses of the bodies after the interaction.

Jet motion is a motion caused by the separation of some of its parts from the body at a certain speed. The basis of jet motion is the law of conservation of momentum. An example of jet motion is the motion of rockets (fuel combustion products are separated).

Problem-solving examples

1. A projectile of mass $25\,kg$, flying horizontally with a speed of $400\,m/s$, hits a sand cart of mass $1875\,kg$ and becomes stuck in the sand. At what speed will the cart move if, before the projectile hit, it was moving at a speed of $2\,m/s$ in the direction of the projectile? At what speed will the cart move if the projectile flies against the cart's motion?

Known quantities:

$m_1 = 25\,kg$
$m_2 = 1875\,kg$
$\upsilon_1 = 400\,m/s$
$\upsilon_2 = 2\,m/s$
$\upsilon - ?\ \upsilon' - ?$

Problem solution

Before the projectile hit the sand, the projectile had momentum $m_1\upsilon_1$, and the cart had momentum $m_2\upsilon_2$. The total momentum of the system is $m_1\upsilon_1 + m_2\upsilon_2$.

After the projectile hits the sand, the total momentum will be equal:

$$(m_1 + m_2)\upsilon .$$

Considering the impact to be completely inelastic in the horizontal projection, we have, according to the law of conservation of momentum

$$m_1\upsilon_1 + m_2\upsilon_2 = \left(m_1 + m_2\right)\upsilon .$$

$$\upsilon = \frac{m_1\upsilon_1 + m_2\upsilon_2}{m_1 + m_2} = \frac{25\cdot 400 + 1875\cdot 2}{1900} = 7{,}2\,m/s$$

In the case when the projectile flies against the movement of the cart, we will get:

$$m_1\upsilon_1 - m_2\upsilon_2 = \left(m_1 + m_2\right)\upsilon' \Rightarrow$$

$$\upsilon' = \frac{m_1\upsilon_1 - m_2\upsilon_2}{m_1 + m_2} = \frac{25\cdot 400 - 1875\cdot 2}{1900} = 3{,}3\,m/s .$$

Answer: $\upsilon = 7{,}2\,m/s$; $\upsilon' = 3{,}3\,m/s$

2. A grenade flying at a speed of $\upsilon = 10\,m/s$, broke into two fragments. The larger fragment, the mass of which was 0,6 of the mass of the entire grenade, continued to move in the former direction, but with an increased speed of $u_1 = 25 m/s$. Calculate the speed u_2 of the smaller fragment.

Known quantities:	*Problem solution*
$\upsilon = 10 m/s$ $m_1 = 0,6m$ $m_2 = 0,4m$ $u_1 = 25 m/s$ u_2-?	The law of conservation of momentum is written in the following form: $m_1\vec{\upsilon}_{01} + m_2\vec{\upsilon}_{02} + ... + m_n\vec{\upsilon}_{0n} = m_1\vec{\upsilon}_1 + m_2\vec{\upsilon}_2 + ... + m_n\vec{\upsilon}_n,$

Let's point the X axis in the direction of movement of the grenade. In the projection onto the horizontal axis, according to the law of conservation of momentum, we have:

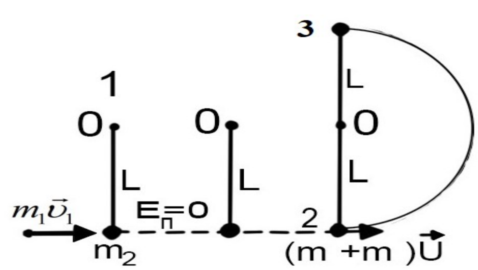
$$m\upsilon = 0,6mu_1 + 0,4mu_2.$$

$$u_2 = \frac{\upsilon - 0,6u_1}{0,4} = \frac{10 - 0,6 \cdot 25}{0,4} = -12,5 m/s.$$

That is, the smaller fragment moves in the opposite direction.

Answer: $u_2 = -12,5 m/s.$

3. A bullet of mass $m_1 = 5 g$ flies horizontally with a speed of $\upsilon_1 = 500 m/s$. It falls into a large ball of mass $m_1 = 0,5 kg$, which hangs on a weightless rod, and gets stuck in it. At what length of the rod L will the large ball rise on the hinged mounting of the rod to the highest point of the trajectory? If the rod is replaced by a non-stretchable rope, should it be longer or shorter than the rod?

Known quantities	
$\upsilon_1 = 500 m/s$, $m_1 = 5 g = 5 \cdot 10^{-3} kg$ $m_1 = 0,5 kg$ $L - ?$	

Problem solution

Let's set the zero level of potential energy at the lowest point of the big sphere. At the highest point, the speed of the bullet is zero. According to the law of conservation of momentum for an inelastic impact (transition from position 1 to position 2), we find the initial velocity υ of the bullet:

$$m_1\upsilon_1 = (m_1 + m_2)\upsilon; \qquad \upsilon = \frac{m_1\upsilon_1}{m_1 + m_2}.$$

At the transition from position 2 to position 3, the system is conservative and closed. According to the law of conservation of mechanical energy, we have:

$$\frac{(m_1 + m_2)\upsilon^2}{2} = (m_1 + m_2)g2L.$$

$$L = \frac{\upsilon^2}{4g} = \frac{1}{4g}\left(\frac{m_1\upsilon_1}{m_1 + m_2}\right)^2 = \frac{1}{4\cdot 10}\left(\frac{0,005\cdot 500}{0,005 + 0,5}\right)^2 = 0,61 \text{ m}.$$

If the rod is replaced by a rope, then the condition for the ball to reach the highest point changes. For the rod, it was the condition of zero velocity, and for the rope, it was the equality of its tension force. Therefore, at the upper point, the ball must have a non-zero velocity. Not all of the kinetic energy at the bottom point is converted into potential energy, so the rope must be shorter than the rod.

Answer: $L = 0,61$ m.

3.2. Energy. Law of conservation of energy

Energy is a single measure of various forms of movement and interaction of matter. Energy characterizes the movement of the system, as well as the interaction of bodies or particles in the system. The SI unit of energy is the joule, J.

Mechanical energy is partially or completely transformed into other types - internal or electromagnetic field energy.

The **law of conservation of energy** - energy does not arise and does not disappear, it only transforms from one form to another and is transferred from one body to another.

Types of energy: 1) mechanical energy of the body (potential and kinetic); 2) internal; 3) electromagnetic (electric + magnetic); 4) chemical; 5) light; 6) nuclear, atomic.

Kinetic energy is the energy of a moving body. The kinetic energy of a body with a mass of m, moving at a speed of υ is determined by the formula: $W_k = \frac{m\upsilon^2}{2}$.

Potential energy is energy due to the interaction of bodies or body particles. In mechanics, the following types of energy are distinguished:

а) potential energy of a body with a mass m, raised above the Earth to a height h:

$$W_p = mgh;$$

б) potential energy of an elastically deformed body:

$$W_p = \frac{kx^2}{2},$$

where k is the stiffness of the body (spring); x **is an extension**.

в) potential energy of the gravitational interaction of two material points with masses m_1 and m_2, located at a distance of r from each other:

$$W_p = \frac{Gm_1m_2}{r}.$$

The **total mechanical energy** of the system is the sum of the kinetic and potential energy of all the bodies of the system:

$$W = W_k + W_p.$$

The **law of conservation of total mechanical energy**: in a closed system of bodies that interact only by elastic forces and gravitational forces, the total mechanical energy remains unchanged (conserved):

$$W = const, \quad W_{k0} + W_{p0} = W_k + W_p.$$

The **change in kinetic energy of the system** is equal to the work done by external forces applied to the system:

$$A = W_{k2} - W_{k1}.$$

Problem-solving examples

1. The stone and the ball are located at a height of $5m$. At the initial moment, the momentum and kinetic energy of the stone are $8kg \cdot m / c$ and $16J$, respectively, and the momentum and kinetic energy of the ball are $8kg \cdot m / c$ and $32J$, respectively. Determine the mass of the stones and the ball, the speed of their movement at the initial moment and at the moment of falling to the ground. Neglect air resistance.

Known quantities:	*Problem solution*
$p_s = p_b = 8kg \cdot m / s$ $W_{ks} = 16J$ $W_{kb} = 32J$ $h = 5m$	According to the condition of the problem, let's write the systems of equations for each body.
$m - ?\ \upsilon_{0s} - ?\ \upsilon_{0b} - ?$ $\upsilon_s - ?\ \upsilon_b - ?$	

For stone: momentum is $m_s \upsilon_s = 8kg \cdot m / s$, kinetic energy is $\dfrac{m_s \upsilon_s^2}{2} = 16J$

For the ball: momentum is $m_b \upsilon_b = 8kg \cdot m / s$, kinetic energy is $\dfrac{m_b \upsilon_b^2}{2} = 32J$.

Solving these systems of equations, we will get the mass of the stone and the mass of the ball: $m_s = 2\,kg$, $m_b = 1\,kg$. The initial speeds, respectively, are equal: $\upsilon_{0s} = 4\,m/s$, $\upsilon_{0b} = 8\,m/s$.

Let's find the formula for the speed of movement of stones and the speed of movement of the ball at the moment of falling to the ground. According to the law of conservation of energy, the mechanical energy of the system is conserved:

$W_k + W_p = const$, $mgh + \frac{m\upsilon_0^{\,2}}{2} = \frac{m\upsilon^2}{2}$, where υ is the speed of movement of the body at the moment of falling to the ground.

Then $mgh = \frac{m\upsilon^{\,2}}{2} - \frac{m\upsilon_0^{\,2}}{2} = \frac{m}{2}\left(\upsilon^2 - \upsilon_0^2\right)$. We solve this equation with respect to the speed and get $\upsilon = \sqrt{2gh + \upsilon_0^2}$.

For stone we get $\upsilon_s = \sqrt{2 \cdot 10 \cdot 5 + 4^2} = 10{,}8\,m/s$,

For the ball we have: $\upsilon_b = \sqrt{2 \cdot 10 \cdot 5 + 8^2} = 12{,}8\,m/s$.

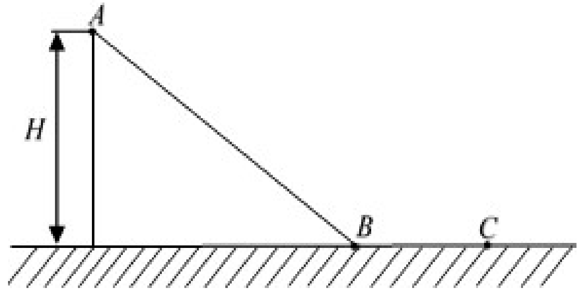
<u>Answer</u>: $m_s = 2\,kg$; $m_b = 1\,kg$; $\upsilon_{0s} = 4\,m/s$, $\upsilon_{0b} = 8\,m/s$;; $\upsilon_s = 10{,}8\,m/s$; $\upsilon_b = 12{,}8\,m/s$.

2. A body with a mass of $50\,kg$ rolls downs an inclined plane from a height of $3\,m$ and stops after passing through a horizontal section of the path. Determine the length of the horizontal section, if the resistance force on the horizontal section is equal to $60\,N$. Neglect the resistance force on the inclined plane.

Known quantities:

$m = 50\,kg$
$F = 60\,N$
$h = 3\,m$

$l - ?$



Problem solution

On an inclined plane, as shown in the figure, the resistance force can be neglected in the section AB. Let's use the law of conservation of total mechanical energy of the body $W = W_k + W_p$.

In the upper A and lower B points of the trajectory, the total mechanical energy of the body is the same. At the top point:

$$W_{tp} = W_k + W_p = 0 + mgh = mgh .$$

At the bottom point: $W_{bp} = mgh$.

A resistance force acts on the horizontal section of the aircraft, so the change in total energy is equal to the work of the resistance force: $mgh = F_r l$. From this formula, we will find an expression for the length of the horizontal section of the path :

$$mgh = F_r l \Rightarrow l = \frac{mgh}{F_r}, \quad l = \frac{50 \cdot 10 \cdot 3}{60} = 25\,m$$

Answer: $l = 25m$

3. A ball is thrown vertically upwards with an initial speed of $10\,m/s$. At what height does the ball's kinetic energy equal its potential energy?

Known quantities:

$\upsilon_0 = 10\,m/s$

$h - ?$

Problem solution

According to the law of conservation of energy, the energy at the initial moment of time is equal to the energy at the next moment

$$W_{k0} + W_{p0} = W_{k1} + W_{p2}.$$

Since from the condition of the problem $W_{k1} = W_{p2}$, we get:

$\frac{m\upsilon_0^2}{2} + 0 = mgh + mgh = 2mgh$. We will find the height from this expression:

$$h = \frac{\upsilon_0^2}{4g} = \frac{10^2}{4 \cdot 9{,}8} \approx 2{,}5m.$$

Answer: $h \approx 2{,}5m.$

3.3. Mechanical work. Power

There are two ways of transferring motion or energy from one macro body to another: in the form of work and in the form of heat (heat exchange). The first way to change energy is called mechanical work.

Work is a measure of change and transformation of energy $A = \Delta W$.

The SI unit of work (as well as energy) is the joule, J: $[A] = [W] = J.$

Mechanical work A is a value that is equal to the product of the modulus of force by the modulus of displacement and the cosine of the angle between force and displacement (Fig. 6): $A = Fs\cos\alpha$.

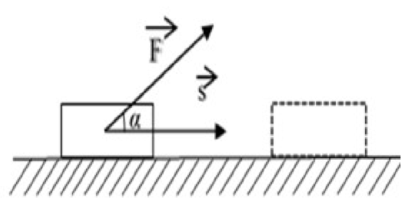


Figure 6.

If $90^0 < \alpha \geq 0$, then $A > 0$ corresponds to an increase in energy;

If $\alpha = 90^0$, then $A = 0$ corresponds to the constancy of energy;

If $90^0 < \alpha \leq 180^0$, then $A < 0$ corresponds to a decrease in energy;

The work of a constant force is maximum, if $\alpha = 0$: $A = Fs$.

Kinetic energy theorem: the work of forces that cause a change in the speed of movement of a body is equal to the change in its kinetic energy:

$$A = W_k - W_{k0} = \frac{m\upsilon^2}{2} - \frac{m\upsilon_0^2}{2}.$$

Potential energy theorem: the work of gravity and elastic force is equal to the change in potential energy taken with the opposite sign:

$$A = -(W_p - W_{p0}).$$

The work of gravity: $A = mgh_0 - mgh = mg(h_0 - h)$.

The work of elastic force: $A = \frac{kx_0^2}{2} - \frac{kx^2}{2}$.

Power characterizes the speed of work. The power is equal to the ratio of the performed work A to the time interval t, during which it was performed: $P = \frac{A}{t}$.

The SI unit of power is watt, W. $[P] = \frac{J}{s} = W$.

The power at constant force is equal to: $P = F\upsilon$.

Problem-solving examples

1. A car moves along a horizontal road with uniform acceleration. The speed of a car with mass 2 tons in time $4s$ increased from $36 km/h$ to $72 km/h$. Neglect the drag force. Calculate the work of the traction force of the car and its power.

Known quantities:
$m = 2\,tons = 2\cdot10^3\,kg$
$\upsilon_0 = 36\,km/h = 10\,m/s$
$\upsilon = 72\,km/h = 20\,m/s$
$t = 4\,s$
$P - ?,\ A - ?$

Problem solution

Since the force of resistance to motion can be neglected, the traction force of the car changes its speed. According to the kinetic energy theorem, we have:

$$A = W_k - W_{k0} = \frac{m\upsilon^2}{2} - \frac{m\upsilon_0^2}{2} = \frac{m}{2}(\upsilon^2 - \upsilon_0^2),$$

$$A = \frac{2\cdot10^3}{2}(20^2 - 10^2) = 300\cdot10^3 = 300\ kJ.$$

The power of the car can be determined by the formula:

$$P = \frac{A}{t},\ P = \frac{300}{4} = 75\,kW.$$

Answer: $A = 300\,kJ$, $P = 75\,kW$.

2. A wagon with an initial speed of $15\,m/s$ moves horizontally. Under the action of the force of friction $F = 6000\,N$ wagon stops after some time. Mass of the wagon is 20 tons. Calculate the braking distance S and the work A of the friction force.

Known quantities:
$m = 20\,tons = 2\cdot10^4\,kg$
$\upsilon = 15\,m/s$
$F = 6000\,N$
$S - ?,\ A - ?$

Problem solution

According to the theorem on the change in kinetic energy: the change in kinetic energy is equal to the work of all external forces, in our case, the work of the friction force, i.e. $A = FS$

$$A = FS = \frac{m\upsilon^2}{2} = \frac{2\cdot10^4\cdot15^2}{2} = 2{,}25\cdot10^6\text{J};\ \Rightarrow$$

$$\text{S} = \frac{m\upsilon^2}{2F} = \frac{2{,}25\cdot10^6}{6000} = 375\,m$$

Answer: $\text{S} = 375\,m$, $A = 2{,}25\cdot10^6\,J$.

3. The conveyor lifts a load of mass $1,2$ tons in time $20s$. The length of the conveyor belt is $8m$, the angle of inclination is 30^0. The conveyor efficiency is equal to 75%. Find the net work, total work and power of the conveyor motor.

Known quantities:

$m = 1,2 tons = 1,2 \cdot 10^3 kg$
$t = 20s$
$l = 8m$
$\alpha = 30^0$
$\eta = 0,75$

$A_{net} - ?\ A_{total} - ?\ P - ?$

Problem solution

When the load is lifted to a height of h by the conveyor, useful work is performed, which is calculated according to the formula:

$$A_{net} = mgh.$$

The height to which the conveyor lifts the load is $h = l\sin\alpha = 8\sin 30^0 = 4m$, where l is the length of the conveyor belt.

Hence, $A_{net} = mgh = 1{,}2\cdot 10^3\cdot 10\cdot 4 = 48\cdot 10^3 J$.

The complete work can be found from the ratio:

$$\eta = \frac{A_{net}}{A_{total}} = 0{,}75 \Rightarrow A_{total} = \frac{A_{net}}{\eta} = \frac{48\cdot 10^3}{0{,}75} = 64\cdot 10^3 J.$$

We calculate the conveyor engine power using the formula:

$$P = \frac{A_{total}}{20} = \frac{64\cdot 10^3}{20} = 3{,}2\cdot 10^3 W.$$

Answer: $A_{net} = 48\cdot 10^3 J$, $A_{total} = 64\cdot 10^3 J$, $P = 3{,}2\cdot 10^3 W$.

3.4. Problems and tests for self-solving on the topic «Laws of conservation in mechanics»

Problem solving guidelines. The use of the law of conservation of mechanical energy significantly simplifies the solution of problems in which two states of the system of interacting bodies are considered and allows not considering the forces acting between the bodies. It is recommended to make a drawing, noting on it the initial and final position of the body. Select the zero reference level of potential energy. The zero level is associated with the lowest value of potential energy. Specify the velocities and coordinates of the body characterizing the state of the body in both positions. Write down the law of conservation of mechanical energy. If there are frictional forces in the system, take them into account. If the system is not closed, take into account the work of external forces. If during the transition of the system of bodies from the initial state to the final state, frictional forces acted on the system or external forces and inelastic deformation forces acted on it, then mechanical energy is not conserved. In this case, the kinetic energy theorem can be used to solve problems.

Problems about dividing one body into parts (or, conversely, about connecting several bodies into one), as well as problems about elastic or inelastic impact of bodies require the application of the law of conservation of momentum. It is necessary to make a drawing on which the momentum vectors at the beginning and at the end of this process are depicted for each body of the system. Write the equations of the law of conservation of momentum of bodies in projections on the OX and OY axes. If the direction of the momentum vector coincides with the positive direction of the coordinate axis or forms an acute angle with it, then the projection of the momentum has a "+" sign, if not, then a "-" sign. Solving a significant number of problems requires the simultaneous use of the law of conservation of momentum and the law of conservation or change of mechanical energy. Since energy is a scalar quantity, and momentum is a vector quantity, the equation of the law of conservation of momentum is written in projections on the selected coordinate axes.

When solving problems about the work of a constant force, it is necessary to determine the work of which force should be determined and write down the initial equation. If the force is not determined by the condition of the task, then it is found from the equation of Newton's second law. When calculating the power developed by a constant force, the formula for the relationship between power and work should be applied.

1. A boy with a mass of $50 kg$, sliding down a hill on a sled, drove along a horizontal road to a stop on route $20 m$ in $10 s$. Determine the force of friction and the coefficient of friction.

2. What was the kinetic energy of a body with a mass of $2\ kg$, if it rose on an inclined plane with an angle of inclination of 60^0 to a height of $1 m$? The coefficient of friction between the body and the inclined plane is $0{,}1$.

3. Calculate the work performed when lifting a load of mass $2\ kg$ along an inclined plane with an angle of inclination of 30^0 for a distance of $2 m$, if the time of lifting the load is $2 s$, and the coefficient of friction is $0{,}1$.

4. A shot was fired vertically upwards from a spring gun. Determine the height to which a bullet with a mass of $15g$, will rise if a spring with a stiffness of $200 N/m$ was compressed before the shot by $10cm$. Neglect the mass of the spring.

5. A ball of mass $1kg$, moving with a speed of $2m/s$, hits a ball of mass $3kg$, moving with a speed of $1,5\ m/s$. Determine the speed of the balls after a perfectly elastic central impact.

6. A projectile of mass $100\ kg$, flying horizontally along a railway track with a speed of $500m/s$, hits a sand wagon of mass 10 tons, and gets stuck in it. What speed will the wagon get if it was stationary before that?

7. At what speed should a hockey puck with a mass of $160\ g$, fly so that its momentum is equal to the momentum of a bullet with a mass of $8g$, flying at a speed of $600m/s$.

8. The momentum of the body is $8kg \cdot m/s$, and the kinetic energy is $16J$. Determine the mass and speed of the body.

9. A body of mass $100g$, thrown with a speed of $10m/s$ from a height of $20m$, fell to the ground with a speed of $20m/s$. Calculate the work done to overcome the force of air resistance.

10. A body with a mass of $250kg$ falls from a height of $800m$. What is its potential and kinetic energy at a height of $100m$ above the surface of the earth and at the moment of falling to the ground? Neglect air resistance.

11. Determine the work that must be done to increase the speed of movement of the body with mass $m = 1kg$ from $2m/s$ to $6m/s$ on the way $10m$ The force of friction $2N$ acts on the entire path.

12. The plane rises to a height of $5km$, while its speed is equal to $\upsilon = 360\,km/h$. How many times the work A_1, that was done during the ascent

against gravity greater than the work A_2, that was done when the speed of the plane increased?

13. What power P does the engine of a car with a mass of 1,5 tons, develop, if it is known that the car travels at a constant speed of $\upsilon = 36\,km/h$ on a horizontal road? The coefficient of friction is $\mu = 0{,}015$.

14. A carriage with a mass of 25 tons, moving uniformly decelerated with an initial speed of $\upsilon_0 = 54\,km/h$, stops after some time under the action of a frictional force of $6\,kN$. Determine the work of frictional forces and the distance the car will travel to stop.

15. What work must be done to raise a load weighing $30\,N$ to a height of $10\,m$ with uniform acceleration for $5\,s$? What power should the engine have for such a lift, if the efficiency of the engine is 80%?

Tests

1. The dependence of the velocity projection on time has the form $\upsilon = 10 + t$ or a body whose mass is $300\,g$. Determine the dependence of the projection of the pulse on time $p_x(t)$.

A	B	C	D
$p_x = 3 + 0{,}3t$	$p_x = 0{,}3 + 3t$	$p_x = 10 + 3t$	$p_x = 300 + 3t$

2. Two electrons move towards each other along the line connecting their centers with velocities of the same magnitude υ. What will be the speed of electrons at their maximum convergence?

A	B	C	D
υ	2υ	$\upsilon/2$	0

3. A force $6N$ acts on a body whose mass is $3\,kg$, and which moves with an initial speed of $-2m/s$. Determine the dependence of the projection of the impulse on time $p_x(t)$.

A	B	C	D
$p_x = 6 - 6t$	$p_x = 6 + 6t$	$p_x = -6 + 6t$	$p_x = -6 - 6t$

4. The laws of change over time of the coordinate and projection of the body's momentum in SI have the form: $x = -5 + 2t - t^2$; $p_x = 1 - t$. Determine the body mass.

A	B	C	D
$100\,g$	$200\,g$	$250\,g$	$500\,g$

5. Specify the physical quantity, the unit of which is determined through the main SI units as $\frac{kg \cdot m^2}{s^3}$.

A	B	C	D
Force work	Power	Body momentum	Force impulse

6. Two balls made of the same substance fall from the same height. The radius of the first sphere is twice as large as the radius of the second. Compare the work of gravity.

A	B	C	D
$A_1 = 8A_2$	$A_1 = 4A_2$	$A_1 = 2A_2$	$A_1 = A_2$

7. A car with a mass of $12\,tons$, which moves in a straight line with a speed of $36\,km/h$, stops after a time of $20\,s$ under the action of resistance forces. What is the average power of resistance forces?

A	B	C	D
$3\,kW$	$4\,kW$	$5\,kW$	$6\,kW$

8. A force of constant magnitude and direction was applied to the body, which brought the body out of a state of rest. How many times does the work of this force on the first half of the path differ from the work on the second half of the path?

A	B	C	D
$A_1 = 2A_2$	$2A_1 = A_2$	$A_1 = A_2$	$A_1 = 3A_2$

9. A force of constant magnitude and direction was applied to the body, which brought the body out of a state of rest. How many times does the work A_1 of this force in the first half of the time differ from the work A_2 in the second half of the time?

A	B	C	D
$A_1 = A_2$	$2A_1 = A_2$	$A_1 = 2A_2$	$A_1 = 3A_2$

10. Two boys whose masses are $60kg$ and $40kg$, are standing next to each other on ice skates. By how much will the boys' kinetic energies differ immediately after they push away from each other?

A	B	C	D
$\frac{E_{k1}}{E_{k2}} = \frac{3}{2}$	$\frac{E_{k1}}{E_{k2}} = \frac{2}{3}$	$\frac{E_{k1}}{E_{k2}} = 1$	$\frac{E_{k1}}{E_{k2}} = \frac{1}{3}$

11. A body whose mass is m, has momentum p. Determine the kinetic energy of this body.

A	B	C	D
$E_k = \frac{2p^2}{m}$	$E_k = \frac{p^2}{m}$	$E_k = \frac{p^2}{2m}$	$E_k = \frac{p^2}{4m}$

12. A dynamometer whose spring is stretched by $4cm$, shows $2N$. Determine the potential energy of the deformed spring.

A	B	C	D
$40\ mJ$	$400\ J$	$0{,}4\ J$	$4\ mJ$

13. A stone thrown on the surface of the ice with a speed of $2m/s$, moved the path $20m$ to a complete stop. Determine the coefficient of friction of the stone on the ice.

A	B	C	D
$0{,}01$	$0{,}02$	$0{,}03$	$0{,}04$

14. With what minimum speed must a ball be thrown down from a height h, so that after a perfectly elastic impact with the ground it bounces to a height $2h$? Ignore air resistance.

A	B	C	D
$\sqrt{gh}$	$\sqrt{2gh}$	$\sqrt{3gh}$	$2\sqrt{gh}$

15. By how many times do the potential energies of a deformed spring differ, if the forces that deform them are equal to $1{,}5N$ and $1N$, respectively.

A	B	C	D
$1{,}5$	$2{,}25$	$2{,}5$	$4{,}5$

4. MECHANICAL OSCILLATIONS

4.1. Harmonic oscillations

Oscillations are movements that are periodically repeated over time.

Harmonic oscillations are oscillations in which the oscillating quantity changes with time according to the law of sine or cosine

$$x = A\cos(\omega t + \varphi_0)$$

where x is the displacement of the point from the equilibrium position.

Oscillation amplitude A is the maximum displacement of a point from the equilibrium position;

The **oscillation period** T is the time during which one complete oscillation is carried out. The SI unit of the oscillation period is a second, s: $[T] = s$.

The **frequency of oscillations** ν is the number of complete oscillations performed by the body per unit of time: $\nu = \frac{1}{T}$. The SI unit of oscillation frequency is hertz, Hz: $[\nu] = \frac{1}{s} = Гц$.

Cyclic frequency ω is a value that is quantitatively equal to the number of complete oscillations in 2π seconds:

$$\omega = \frac{2\pi}{T}$$

Oscillation phase $(\omega t + \varphi_0)$ characterizes the state of the oscillating system at a given moment in time. φ_0 is the initial phase of oscillations at the instant of time $t = 0$.

The speed of movement of a body performing harmonic oscillations is the first derivative of the coordinate in time:

$$\upsilon = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi_0)$$

The acceleration of body movement performing harmonic oscillations is the first derivative of the velocity with time, or the second derivative of the coordinate with time:

$$a_x = \frac{d\upsilon}{dt} = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \varphi_0) = -\omega^2 x$$

The force under which a body of mass m carries out harmonic oscillations, is directly proportional to the absolute displacement of the body from the equilibrium position and is directed opposite to the displacement:

$$F_x = m a_x = -m A\omega^2 \cos(\omega t + \varphi_0) = -m\omega^2 x$$

Spring pendulum is an oscillating system in which oscillations occur under the influence of elastic forces.

Oscillation period of a spring pendulum:

$$T = 2\pi\sqrt{\frac{m}{k}},$$

where m is body mass, k is the stiffness of the spring.

Mathematical pendulum is a material point that is suspended on a weightless, inextensible thread and oscillates under the influence of gravity.

Period of free oscillations of a mathematical pendulum:

$$T = 2\pi\sqrt{\frac{l}{g}},$$

where l is the length of the string of the mathematical pendulum; g is the acceleration of free fall.

Kinetic and potential energy of an oscillating system:

$$E_\kappa = \frac{1}{2}m\upsilon^2 = \frac{m}{2}A^2\omega^2\sin^2(\omega t + \varphi_0)$$

$$E_n = \frac{m}{2}A^2\omega^2\cos^2(\omega t + \varphi_0)$$

The total mechanical energy of the oscillating point:

$$E = E_\kappa + E_n = \frac{1}{2}m\omega^2 A^2.$$

Problem-solving examples

1. The point carries out a harmonic oscillation with a period of $2\,s$ and an amplitude of $50\,mm$. The initial phase of the oscillation is zero. Find the speed of the point at the time when its displacement from the equilibrium position is equal to $25\,mm$.

Known quantities:
$T = 2\,s$
$A = 50\,mm = 0,05\,m$
$x = 25\,mm = 0,025\,m$
$\upsilon - ?$

Problem solution

We write down the equation of oscillation of the point:

$x = 0,05\cos\pi t$, because

$$\omega = \frac{2\pi}{T} = \pi .$$

The speed of a point at a moment in time *t:*

$$\upsilon = \dot{x} = -0,05\pi\sin\pi t$$

The instant of time when the displacement is $0,025\,m$:

$$0,025 = 0,05\cos\pi t_1 ,$$

hence $\cos\pi t_1 = \frac{1}{2}$, $\pi t_1 = \frac{\pi}{3}$. We substitute this value in the expression for speed:

$$\upsilon = -0,05\pi\sin\frac{\pi}{3} = -0,05\pi\frac{\sqrt{3}}{2} = 0,136\,m/s$$

Answer: $\upsilon = 0,136\,m/s$.

2. A material point carries out harmonic oscillations with an amplitude of $A = 2\,cm$. Determine the displacement from the equilibrium position of the oscillating point on which the force $F = 2,25\cdot 10^{-5}\,N$ acts. The total energy is $E = 3\cdot 10^{-7}\,J$.

Known quantities:
$A = 2\,cm = 0,02\,m$
$E = 3\cdot 10^{-7}\,J$
$F = 2,25\cdot 10^{-5}\,N$
$x - ?$

Problem solution

The total energy of a point performing harmonic oscillations is:

$$E = \frac{mA^2\omega^2}{2}$$

The modulus of elastic force is expressed by the displacement of points from the equilibrium position x in the following way $F = kx$.

The circular frequency is: $\omega^2 = \dfrac{k}{m}$,

from here $k = m\omega^2$ and $F = m\omega^2 x$. Having expressed $m\omega^2$ from the formula for energy, we substitute it in the formula for force:

$$m\omega^2 = \frac{2E}{A^2}, \quad F = \frac{2E}{A^2}x.$$

From here we get the expression for the displacement: $x = \dfrac{A^2 F}{2E}$.

Substitution of numerical values gives:

$$x = \frac{4\cdot 10^{-4}\cdot 2{,}25\cdot 10^{-5}}{2\cdot 3\cdot 10^{-7}} = 1{,}5\cdot 10^{-2}\,m.$$

Answer: $x = 1{,}5\cdot 10^{-2}\,m$.

3. Determine the period of oscillation of an elastic pendulum, if the spring in a vertical position is stretched under the action of a weight suspended from it by $1cm$.

Known quantities:

$x = 1cm = 0{,}01m$

$T - ?$

Problem solution

The period of oscillation of the load on the spring is:

$$T = 2\pi\sqrt{\frac{m}{k}},$$

where k is the stiffness of the spring. Since the force of elasticity is equal to the force of gravity:

$$mg = kx \Rightarrow k = \frac{mg}{x}$$

The period of oscillations is found by substituting k into the formula for the period:

$$T = 2\pi\sqrt{\frac{mx}{mg}} = 2\pi\sqrt{\frac{x}{g}}$$

$$T = 2 \cdot 3{,}14\sqrt{\frac{0{,}01}{9{,}8}} = 0{,}2\,s$$

Answer: $T = 0{,}2\,s$.

4.2. Problems and tests for self-solving on the topic «Mechanical oscillations»

Problem solving guidelines. When solving oscillation problems, it is recommended to write down the equation given in the problem and the equation of harmonic oscillations in a general form. Then compare these equations and determine the main characteristics (displacement, amplitude, period, frequency, and phase) according to the task condition. The speed and acceleration of a material point during harmonic oscillations, as well as the maximum value of these values, can be determined from the equation of harmonic oscillations, the parameters of which correspond to the problem data. It is also possible to determine the acceleration modulus of an oscillating point using Newton's second law. If the period of oscillations must be determined in the problem, then the appropriate formulas for the period of harmonic oscillations of elastic or mathematical pendulums should be used. The solution of the obtained equations is reduced to mathematical statements.

1. The maximum kinetic energy of a body on a spring with a stiffness of $200\,N/m$, is equal to $10\,mJ$. Determine the amplitude of the oscillations.

2. Harmonic oscillations of a spring pendulum are described by the sine law. Determine the potential energy of the pendulum at phase $\pi/3$, if the total energy of the pendulum is $40\,mJ$.

3. Determine the ratio of the lengths of two mathematical pendulums, if the ratio of the periods of their oscillations is equal to 1,5.

4. A material point oscillates harmonically according to the law $x = A\cos(\omega t + \varphi_0)$ with a frequency of $0,5\ s^{-1}$. The amplitude of oscillations is $0,03\ m$. Determine the speed of the point at the time when its displacement is $15\ mm$.

5. A material point whose mass is $5\ g$, oscillates harmonically with a frequency of $0,5\ s^{-1}$. The amplitude of oscillations is $3\ cm$. Determine the maximum force acting on the point.

6. The point performs harmonic oscillations, the equation of which has the form $x = 0,05\sin\frac{\pi}{5}t$. At the moment when the point had a potential energy of $0,1\ mJ$, a turning force of $8\ mN$ acted on it. Determine this moment of time.

7. The material point carries out harmonic oscillations so that at the initial moment of time the displacement is $4\ cm$, and the speed is $0,1 m/s$. Determine the speed of the point at the moment when its acceleration is $2s$.

8. The material point oscillates harmonically with a frequency of $0,5\ s^{-1}$. The amplitude of the oscillation is $5 cm$, and the initial phase is $\varphi = 0$. Determine the speed that a point has at the moment when its acceleration is $4 cm/s^2$.

9. A material point oscillates according to law $x = A\sin(\pi t + \frac{\pi}{8})$. At what point in time is the ratio of its kinetic energy to potential energy equal to one?

10. The point oscillates according to the law $x = A\sin\omega t$. At a certain point in time, the displacement of the point turned out to be equal to $5 cm$. When the phase of the oscillations doubled, the displacement became equal to $8 cm$. Determine the amplitude of the oscillations.

11. The point performs harmonic oscillations. The maximum displacement of the point is $10 cm$, and the maximum speed is $20 cm/s$. Determine the cyclic frequency of oscillations and the maximum acceleration of the point.

12. The initial phase of harmonic oscillation is $\varphi = 0$. When the point is displaced from the equilibrium position by $2{,}4\,cm$, the speed of the point is equal to $3\ cm/s$, and when it is displaced by $2{,}8\,cm$ its speed is equal to $2\ cm/s$. Determine the amplitude and period of this oscillation.

13. The point carries out a harmonic oscillation. The oscillation period is $2\,s$, the amplitude is $5\,cm$, the initial phase is zero. Determine the speed υ at the time when the displacement of the point from the equilibrium position is $2{,}5\ cm$.

14. A ball with a mass of $200\,g$ is suspended on a spring and oscillates with a frequency of $5\,Hz$. Determine the stiffness coefficient of the spring.

15. How many times does the oscillation period of a mathematical pendulum change when it is moved from the pole to the Earth's equator?

Tests

1. The length of the suspension of the mathematical pendulum was reduced by 19%. By how many percent did the oscillation period change?

A	B	C	D
19%	81%	10%	20%

2. The amplitude of harmonic oscillations of the point is $5\,cm$, its maximum speed is $4\pi\,cm/s$. Determine the period of oscillations of the point.

A	B	C	D
$0{,}8\,s$	$2{,}5\,s$	$1{,}2\,s$	$2{,}4\,s$

3 When passing through the equilibrium position, the speed of the weight of the spring pendulum is equal to $1\,m/s$. The stiffness of the pendulum spring is $100\ N/m$, the mass of the weight is $10\,g$. Determine the amplitude of oscillations of the pendulum.

A	B	C	D
$5cm$	$3cm$	$2{,}5cm$	$2cm$

4. How will the oscillation period of the mathematical pendulum change when the mass of the burden is doubled?

A	B	C	D
$T_1 = T_2$	$T_1 = \sqrt{2}T_2$	$\sqrt{2}T_1 = T_2$	$T_1 = 2T_2$

5. A weight suspended from a spring at rest stretches it by $2{,}5cm$. Determine the oscillation period of the spring pendulum.

A	B	C	D
π	$\pi/10$	$\pi/2$	$\pi/5$

6. How are the lengths of mathematical pendulums related, if in the same time the first pendulum makes 10 oscillations, and the second 4 oscillations?

A	B	C	D
$\frac{l_2}{l_1} = 1$	$\frac{l_2}{l_1} = 2{,}5$	$\frac{l_2}{l_1} = \sqrt{2{,}5}$	$\frac{l_2}{l_1} = 6{,}25$

7. Determine the path of a material point performing harmonic oscillations over a period. The amplitude of oscillations is $1cm$.

A	B	C	D
$1cm$	$2cm$	$4cm$	$6cm$

8. Determine the modulus of displacement of a material point performing harmonic oscillations per period. The amplitude of oscillations is $1cm$.

A	B	C	D
$1cm$	$2cm$	$4cm$	0

9. A material point oscillates according to the law $x = 0{,}15\cos 0{,}45\pi t$, m. What is the amplitude and period of oscillations?

A	B	C	D
$0{,}15\ m;\ 5s$	$0{,}15\ m;\ 8s$	$15\ m;\ 0{,}8s$	$0{,}25\ m;\ 15s$

10. A load suspended from a spring at rest stretches it by $10cm$. Determine the cyclic frequency of such a spring pendulum.

A	B	C	D
$\pi\ s^{-1}$	$2\pi\ s^{-1}$	$10\ s^{-1}$	$0{,}1\ s^{-1}$

11. Determine the modulus of acceleration of the mathematical pendulum at the moment when it is in the extreme position, in which the angle of deviation from the equilibrium position is $\pi/6$.

A	B	C	D
g	$2g$	$g/2$	0

12. What is the ratio of the lengths of the pendulums on Earth and on the Moon, if the acceleration of free fall on Earth is 6 times greater than on the Moon?

A	B	C	D
$\frac{l_E}{l_M} = 6$	$\frac{l_E}{l_M} = \frac{1}{6}$	$\frac{l_E}{l_M} = \sqrt{6}$	$\frac{l_E}{l_M} = 36$

13. If the mass of the load suspended from the spring is increased by $0{,}1kg$, then the oscillation period of the pendulum increases $\sqrt{1{,}5}$ times. Determine the initial weight of the load.

A	B	C	D
$0{,}1kg$	$0{,}15kg$	$0{,}2kg$	$0{,}25kg$

14. The pendulum of a wall clock oscillates with a frequency of $2\ Hz$. How many times per minute does the potential energy of the pendulum reach its minimum value?

A	B	C	D
30	60	120	240

15. The pendulum of a wall clock oscillates with a frequency of $2\ Hz$. How many times per minute does the potential energy of the pendulum reach its maximum value?

A	B	C	D
30	60	120	240

5. ELEMENTS OF RELATIVITY THEORY

5.1 Elements of special relativity

Length l of a body moving with speed υ relative to some reference system is related to the length l_0 of a body stationary in this system by the ratio:

$$l = l_0\sqrt{1-\beta^2},$$

where $\beta = \frac{\upsilon}{c}$, c is the speed of light in a vacuum.

Time interval τ' in the system moving with speed υ relative to the observer is related to the time interval τ in the stationary system for the observer by the ratio:

$$\tau' = \frac{\tau}{\sqrt{1-\beta^2}}.$$

Dependence of the mass m of the body on the speed of its movement has the form:

$$m = \frac{m_0}{\sqrt{1-\frac{\upsilon^2}{c^2}}},$$

where m_0 is the rest mass of the body.

Dependence of the kinetic energy of the body on the speed of its movement:

$$E_к = m_0 c^2 \left(\frac{1}{\sqrt{1 - \frac{\upsilon^2}{c^2}}} - 1 \right).$$

The **law of the relationship of mass and energy:**

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}.$$

A change in the mass of the system by the amount Δm corresponds to a change in the energy of the system by the amount:

$$\Delta E = c^2 \Delta m.$$

Relativistic dependence between total energy and momentum of a particle:

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}.$$

If the body is stationary, then $E_0 = m_0 c^2$, where E_0 is the rest energy of the body.

Problem-solving examples

1. What speed should a moving body have so that its ultimate size is reduced by 2 times?

Known quantities:	Problem solution
$l = \frac{l_0}{2}$	A body moves at a constant speed relative to an inertial frame of reference. In this system, the length of the body is equal:
$\upsilon - ?$	

$$l = l_0\sqrt{1-\beta^2}\,,\quad \beta = \frac{\upsilon}{c}.$$

According to the condition of the problem: $l = \frac{l_0}{2}$. Then $\frac{l_0}{2} = l_0\sqrt{1-\beta^2}$,

$$\sqrt{1-\beta^2} = \frac{1}{2}.$$

Hence $\beta^2 = 0,75$. $\frac{\upsilon}{c} = \sqrt{0,75}$.

$$\upsilon = c\sqrt{0,75} = 3\cdot 10^8\sqrt{0,75} = 2,6\cdot 10^8\, m/s.$$

<u>Answer:</u> $\upsilon = 2,6\cdot 10^8\, m/s$.

2. Determine the relativistic momentum and kinetic energy of an electron moving at a speed of $\upsilon = 0,9c$ (c is the speed of light in a vacuum).

<u>Known quantities:</u>

$\upsilon = 0,9c$

$p - ?,\ E_\kappa - ?$

<u>Problem solution</u>

The relativistic momentum is determined by the formula:

$$p = m_0 c\frac{\beta}{\sqrt{1-\beta^2}} = 9{,}11\cdot 10^{-31}\cdot 3\cdot 10^8\,\frac{0{,}9}{\sqrt{1-0{,}9^2}} = 5{,}6\cdot 10^{-22}\,\frac{kg\cdot m}{s}.$$

The kinetic energy of a particle is defined as the difference between the total energy E and the rest energy E_0 of this particle: $E_\kappa = E - E_0$,

$$E = mc^2,\quad E_0 = m_0c^2$$

$$E_k = \frac{m_0c^2}{\sqrt{1-\frac{\upsilon^2}{c^2}}} - m_0c^2 = m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) =$$

$$= 9{,}11\cdot 10^{-31}\cdot 9\cdot 10^{18}\cdot\left(\frac{1}{\sqrt{1-0{,}9^2}} - 1\right) = 1{,}06\cdot 10^{-13}\,\text{J}.$$

Answer: $p = 5{,}6 \cdot 10^{-22} \dfrac{kg \cdot m}{s}$, $E_k = 1{,}06 \cdot 10^{-13}$ J.

3. Determine the speed of a meson if its total energy is 10 times its rest energy.

Known quantities:

$\dfrac{E}{E_0} = 10$

$\upsilon - ?$

Problem solution

The total energy of a meson consists of its kinetic energy and rest energy.

Kinetic energy of the body: $E_\kappa = m_0 c^2 \left(\dfrac{1}{\sqrt{1 - \dfrac{\upsilon^2}{c^2}}} - 1 \right)$,

The rest energy E_0 of the body is: $E_0 = m_0 c^2$

Then the total energy is: $E = \dfrac{m_0 c^2}{\sqrt{1 - \dfrac{\upsilon^2}{c^2}}}$.

Under the condition of the problem: $\dfrac{E}{m_0 c^2} = 10$. $\dfrac{1}{\sqrt{1 - \dfrac{\upsilon^2}{c^2}}} = 10$.

From here $\dfrac{\upsilon}{c} = 0{,}995$.

$\upsilon = \beta \cdot c = 2{,}985 \cdot 10^8 m/s$

Answer: $\upsilon = 2{,}985 \cdot 10^8 m/s$.

5.2. Problems and tests for self-solving on the topic «Elements of relativity theory»

Problem solving guidelines: When solving problems on relativistic effects, it is customary to write not in km/h or km/s, but in fractions of the speed of light. When solving the problems of this section, it is necessary to first of all imagine what values are recorded in which frame of reference (eigenvalues of length, mass, time are given for the frame of reference moving with the body). Many problems are solved more easily when using invariant relations. It must be remembered that the concept of simultaneity of events is relative.

1. The intrinsic lifetime of a particle differs by 1.5% from the lifetime according to a stationary clock. Determine $\beta = \frac{\upsilon}{c}$.

2. Determine the speed at which the relativistic momentum of a particle exceeds its Newtonian momentum by 5 times.

3. Determine the relativistic momentum of an electron whose kinetic energy is $1GeV$.

4. An electron moves with a speed of $\upsilon = 0,5c$. Determine the kinetic energy of an electron.

5. An electron moves with a speed of $\upsilon = 0,6c$. Determine the momentum of the electron.

6. The kinetic energy of the electron is $1MeV$. Determine the speed of the electron.

7. What speed should a moving body have so that its longitudinal dimensions decrease by a factor of 2?

8. Cosmic ray mesons reach the Earth's surface with a wide variety of velocities. Calculate the relativistic contraction of a meson that has a speed equal to 95% of the speed of light.

9 By how much will the mass of a α-particle increase when it is accelerated from its initial speed equal to zero to a speed of $\upsilon = 0,9c$, (where c is the speed of light)?

10. At what speed is the mass of a moving electron twice its rest mass?

11. Determine the speed of a meson if its total energy is 10 times its rest energy.

12. What fraction of the speed of light must be the speed of a particle so that its kinetic energy is equal to its rest energy?

13. The synchrophasotron produces a beam of protons with a kinetic energy of $10^4\, MeV$. What fraction of the speed of light is the speed of protons in this beam?

14. Electrons flying out of the cyclotron have a kinetic energy of $0,67\, MeV$. What fraction of the speed of light is the speed of these electrons?

15. The mass of a moving electron is twice its rest mass. Calculate the kinetic energy of this electron.

Tests

1. At what speed should a proton whose mass is 1 atomic mass unit, move so that its relativistic mass becomes equal to the rest mass of α - particle equal to 4 atomic mass unit (c is the speed of light)?

A	B	C	D
$0,67c$	$0,77c$	$0,87c$	$0,97c$

2. What physical quantity remains unchanged in moving and stationary inertial frames of reference?

A	B	C	D
Body energy	Body charge	Body momentum	Body mass

3. What physical quantity in the frame of reference relative to which the body is stationary is greater than in the frame of reference relative to which the body is moving?

A	B	C	D
Body energy	Body charge	Body momentum	Segment length

4. A light source moves towards a stationary observer with a speed of $\upsilon = 0{,}5c$. What is the speed of light relative to the observer?

A	B	C	D
0	$0{,}5c$	c	$1{,}5c$

5. An observer moves past a light source with a speed υ. The speed of light in the frame of reference associated with the observer is...

A	B	C	D
$c+\upsilon$	$c-\upsilon$	c	$\sqrt{c^2+\upsilon^2}$

6. The laws of classical mechanics are valid in those inertial frames of reference relative to which macroscopic bodies move at a speed …

A	B	C	D
$c=\upsilon$	$c<\upsilon$	$c>\upsilon$	$\upsilon-$ is an arbitrary value

7. How many times should time slow down for an observer on Earth on a rocket moving at a speed of $\upsilon = 0{,}8c$?

A	B	C	D
5 / 3	4 / 3	2 / 3	1 / 3

8. Two electrons are moving towards each other with velocities of $\upsilon_1 = 0{,}6c$ and $\upsilon_2 = 0{,}8c$ relative to the Earth. At what speed does the distance between them decrease relative to the Earth?

A	B	C	D
$0,946c$	$0,2c$	c	$1,4c$

9. Two elementary particles fly apart in a vacuum in opposite directions, the speed of each is equal to the speed of light c. What will be the mutual speed of their distance?

A	B	C	D
$0,74c$	$2c$	c	$0,5c$

10. What is the length of a meter rod (for an earthly observer) moving at a speed of $0,6c$?

A	B	C	D
$0,8m$	$0,9m$	$1m$	$1,2m$

11. At what speed relative to the Earth should a spaceship move so that its longitudinal dimensions for an earthly observer are 2 times smaller than the real ones?

A	B	C	D
$0,97c$	$0,87c$	c	$0,6c$

12. A spaceship of length $300m$ is moving away from a stationary observer at a speed of $0,8c$. What is the length of the ship relative to the observer?

A	B	C	D
$180m$	$280m$	$500m$	$580m$

13. How many times will the relativistic mass of a proton change when it passes through a potential difference of $1MV$? The initial speed of the proton is 0.

A	B	C	D
2,95	3,95	1,95	1

14. A body of which mass corresponds to a rest energy of $9 \cdot 10^{13} J$?

A	B	C	D
$1g$	$10g$	$100g$	$1kg$

15. When moving at a certain speed, the longitudinal dimensions of the body halved. How many times has the body weight changed?

A	B	C	D
3	$1,5$	2	4

MOLECULAR PHYSICS AND THERMODYNAMICS

6. MOLECULAR PHYSICS

6.1. Fundamentals of the molecular-kinetic theory of an ideal gas

An **ideal gas** is a physical model of a gas that satisfies such conditions:

1) the own volume of gas molecules is negligibly small compared to the volume of the vessel;

2) there are no interaction forces between gas molecules;

3) collisions of gas molecules with each other and with the walls of the vessel are absolutely elastic.

1 mole is the amount of substance containing the same number of particles as there are in 12 g of carbon ${}^{12}_{6}C$.

Avogadro constant is a value that is numerically equal to the number of particles in 1 mole of any substance: $N_A = 6{,}02 \cdot 10^{23} \dfrac{1}{mol}$.

Molar mass M is the mass of one mole of a substance:

$$M = m_0 N_A = M_r \cdot 10^{-3} \frac{kg}{mol},$$

where m_0 is the mass of a molecule (atom) of the substance under study, M_r – relative atomic mass of a chemical element, contained in Mendeleev's table.

The **amount of a substance** ν is the number of moles in a given mass of the substance:

$$\nu = \frac{m}{M} = \frac{N}{N_A},$$

where N is the number of particles in a given mass of the substance under study.

The basic equation of the molecular-kinetic theory of an ideal gas: $p = \frac{1}{3} n m_0 \overline{\upsilon}^2_{m.sq}$,

where m_0 is the mass of a molecule (atom) of the substance under study, n – is the concentration of molecules, $\overline{\upsilon}^2_{m.sq}$ is the root mean square speed of the molecules. This equation links micro and macro parameters.

If $\overline{E}_k = \frac{m_0 \overline{\upsilon}^2_{m.sq}}{2}$ is the average kinetic energy of translational motion of a molecule, then the basic equation of the molecular kinetic theory can be written in the form:

$$p = \frac{2}{3} n \frac{m_0 \overline{\upsilon}^2_{m.sq}}{2} = \frac{2}{3} n \overline{E}_k \text{ or } p = \frac{1}{3} \rho \overline{\upsilon}^2_{m.sq},$$

where $\rho = \frac{m}{V} = \frac{N m_0}{V} = n m_0$ is the density of the gas, $m = N m_0$ is the mass of the gas.

Temperature is a physical quantity determined by the average kinetic energy $\overline{E}_k$ of translational motion of the particles that make up the system:

$$\overline{E}_k = \frac{3}{2} kT,$$

where $k = 1{,}38 \cdot 10^{-23} J/K$ is **Boltzmann's constant**.

The main equation of the molecular kinetic theory can be rewritten differently:

$$p = \frac{2}{3} n \overline{E}_k = \frac{2}{3} n \cdot \frac{3}{2} kT = nkT,$$

$$p = nkT.$$

Two temperature scales are used: in degrees Kelvin and degrees Celsius. The SI unit of the thermodynamic temperature scale is the Kelvin, *K*. $[T] = K$.

The formula for the relationship between thermodynamic temperature (T) and temperature (t) on the Celsius scale:

$$T = (273 + t)K.$$

From the definition of kinetic energy $\overline{E}_k = \dfrac{m_0 \overline{\upsilon}_{m.sq}^2}{2}$ and the formula for the relationship between kinetic energy and temperature $\overline{E}_k = \dfrac{3}{2}kT$ we obtain a formula for calculating the mean square speed of the translational motion of molecules of an ideal gas:

$$\frac{m_0 \overline{\upsilon}_{m.sq}^2}{2} = \frac{3}{2}kT \Rightarrow \overline{\upsilon}_{m.sq}{}^2 = \frac{3kT}{m_0} \Rightarrow \overline{\upsilon}_{m.sq} = \sqrt{\frac{3kT}{m_0}}.$$

Since $m_0 = \dfrac{M}{N_A}$, therefore $\overline{\upsilon}_{m.sq} = \sqrt{\dfrac{3kN_A T}{M}}$.

Therefore, the **mean square speed of translational motion of molecules** is determined by the following formula:

$$\overline{\upsilon}_{m.sq} = \sqrt{\frac{3RT}{M}},$$

where $R = 8{,}31\dfrac{J}{mol \cdot K}$ is the **universal gas constant**, equal to the product of Boltzmann's constant and Avogadro's constant: $R = kN_A$.

Problem-solving examples

1. Determine the concentration of hydrogen in the cylinder if it is under a pressure of $2{,}7 \cdot 10^5 Pa$, and the mean averege square speed is $2400\, m/s$.

Known quantities:

$M(H_2) = 2 \cdot 10^{-3}\, kg/mol$

$p = 2{,}7 \cdot 10^5\, Pa$

$\bar{\upsilon}_{m.sq} = 2400\, m/s$

$n - ?$

Problem solution

The concentration of molecules is related to pressure by the formula:

$p = nkT$, from where: $n = \dfrac{p}{kT}$.

The unknown temperature can be found from the expression for the mean square velocity: $\bar{\upsilon}_{m.sq} = \sqrt{\dfrac{3RT}{M}}$, $T = \dfrac{\bar{\upsilon}_{m.sq}{}^2 M}{3R}$.

Substituting this expression for temperature into the formula for concentration, we get:

$$n = \frac{3Rp}{k\bar{\upsilon}_{m.sq}^2 M} = \frac{3 \cdot 8{,}31 \cdot 2 \cdot 10^5}{1{,}38 \cdot 10^{-23} \cdot 2{,}4^2 \cdot 10^6 \cdot 2 \cdot 10^{-3}} = 3{,}13 \cdot 10^{25}\, m^{-3}.$$

Answer: $n = 3{,}13 \cdot 10^{25}\, m^{-3}$.

2. Find the amount of substance contained in an aluminum bar weighing 5,4 kg.

Known quantities:

$m = 5{,}4\, kg$

$M = 27 \cdot 10^{-3}\, \dfrac{kg}{mol}$

ν - ?

Problem solution

$$\nu = \frac{m}{M}$$

$$\nu = \frac{5{,}4}{27 \cdot 10^{-3}} = 200\, mol.$$

Answer: 200 mol.

3. At what temperature is the average kinetic energy of translational motion of gas molecules equal to $6,21\cdot 10^{-21}\ J$?

Known quantities:

$\langle W_k \rangle = 6,21\cdot 10^{-21}\ J$

T - ?

Problem solution

The average kinetic energy of translational motion of molecules is determined by the formula:

$$\langle W_k \rangle = \frac{3}{2}kT.$$

From here the temperature is equal:

$$T = \frac{2\langle W_k \rangle}{3k},$$

$$T = \frac{2\cdot 6,21\cdot 10^{-21}}{3\cdot 1,38\cdot 10^{-23}} = 300\ \text{K}.$$

Answer: 300 K.

4. Find the pressure of the gas if the root mean square velocity of the gas molecules is $500\ m/\text{s}$, and its density is $1,35\ kg/\text{m}^3$.

Known quantities:

$\bar{\upsilon}_{m.sq} = 500\ m/\text{s}$

$\rho = 1,35\ kg/\text{m}^3$

P - ?

Problem solution

According to the main equation of the molecular kinetic theory of gases:

$$p = \frac{2}{3}n\frac{m_0\bar{\upsilon}^2_{m.sq}}{2} = \frac{2}{3}n\bar{E}_k \quad \text{or}$$

$$p = \frac{1}{3}\rho\bar{\upsilon}^2_{m.sq},$$

where $\rho = \frac{m}{V} = \frac{Nm_0}{V} = nm_0$ is the density of the gas, $m = Nm_0$ is the mass of the gas.

$$P = \frac{1}{3} \cdot 1,35 \cdot 500^2 = 1,125 \cdot 10^5\, Pa.$$

Answer: 0,11 MPa.

5. Determine the molar mass M of a mixture of oxygen with a mass of $25\, g$ and nitrogen with a mass of $75\, g$.

Known quantities:

$m_1 = 25\, g = 25 \cdot 10^{-3}\, kg$

oxygen O_2

$m_2 = 75\, g = 75 \cdot 10^{-3}\, kg$

nitrogen N_2

$M - ?$

Problem solution

The ratio of the molar mass of the mixture m to the amount of the substance of the mixture in moles ν is:

$$M = \frac{m}{\nu} = \frac{m_1 + m_2}{\frac{m_1}{M_1} + \frac{m_2}{M_2}}.$$

Molar masses of oxygen M_1 and nitrogen M_2:

$$M_1 = 32 \cdot 10^{-3}\, kg/mol,\, M_2 = 28 \cdot 10^{-3}\, kg/mol.$$

Let's do the calculations:

$$M = \frac{(25 + 75) \cdot 10^{-3}}{25 \cdot 10^{-3} / (32 \cdot 10^{-3}) + 75 \cdot 10^{-3} / 28 \cdot 10^{-3}} = 28,9 \cdot 10^{-3}\, kg/mol$$

Answer: $M = 28,9 \cdot 10^{-3}\, kg/mol.$

6.2. Ideal gas laws

The **equation of state of an ideal gas (Mendeleev-Clapeyron equation)**, which relates the macro parameters of one gas state, is defined by the equation:

$$pV = \frac{m}{M}RT,$$

where p is the gas pressure, V is its volume, T is the absolute temperature, m is the mass of the gas, M is the molar mass of the gas, $R = 8{,}31\frac{J}{mol \cdot K}$ is a universal gas constant, $\frac{m}{M} = \nu$ is the amount of substance (number of moles). p, V, T are the macroscopic parameters of the gas state.

A **thermodynamic process** is the transition of an ideal gas from one state to another, which is accompanied by a change in gas parameters.

If the amount of substance does not change during the thermodynamic process, then the consequence of the equation of state is the **Clapeyron equation:**

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} = const.$$

This equation connects the macroscopic states of the system of a given mass of gas during the transition from state 1 to state 2.

An **isoprocess** is a thermodynamic process that occurs with a constant amount of substance, in which one of the macroscopic parameters (p, V, T) remains unchanged.

An **isothermal process** is an isoprocess that occurs at a constant temperature.

An **isobaric process** is an isoprocess that occurs at constant pressure.

An **isochoric process** is an isoprocess that occurs at a constant volume.

Boyle-Marriott's law (isothermal process): for a fixed mass of gas at a constant temperature, the product of gas pressure by its volume is a constant value:

$$p_1V_1 = p_2V_2 \quad \text{or} \quad pV = const,$$

$$T = const,\ \Delta T = 0,\ m = const$$

Graphs of an isothermal process are called **isotherms**. In pV-coordinates the graph of the isotherm is a hyperbola (Fig. 7).

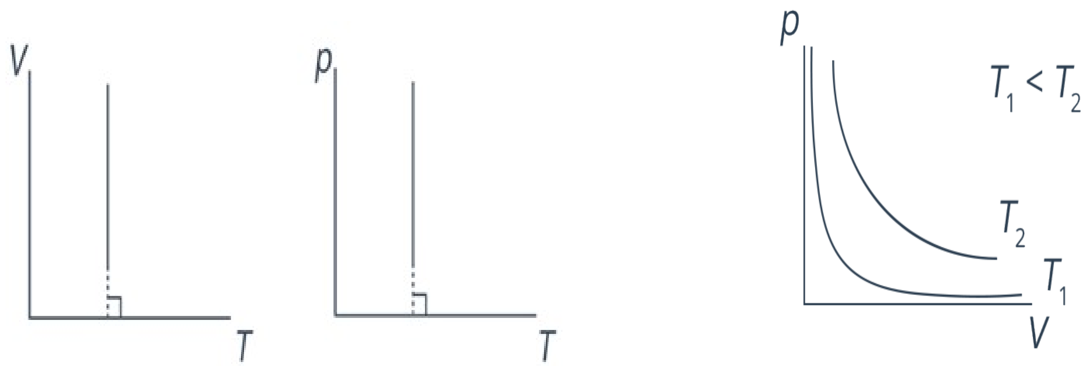


Figure.7. Graphs of isotherms in VT, pT, pV coordinates at different temperatures.

Gay-Lussac's law (isobaric process): for a fixed mass of gas, the ratio of volume to temperature is constant if the pressure of the gas does not change:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = const, \text{ or } \frac{V}{T} = const,$$

$$p = const,\ \Delta p = 0,\ m = const.$$

Graphs of the isobaric process are called **isobars** (Fig. 8).

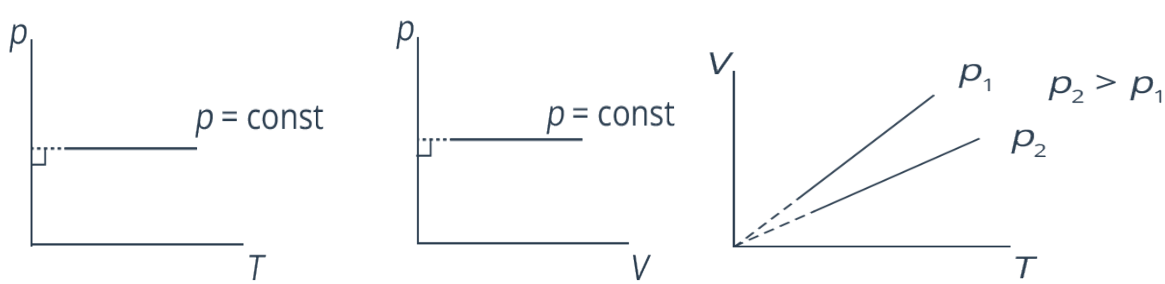


Figure.8. Graphs of isobars in pT, pV, VT coordinates.

Charles' law (isochoric process): for a fixed mass of gas, the ratio of gas pressure to its temperature is constant if the gas volume does not change:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} = const \text{ or } \frac{p}{T} = const,$$

$$V = const,\ \Delta V = 0,\ m = const.$$

Graphs of the isochoric process are called **isochoric** (Fig. 9).

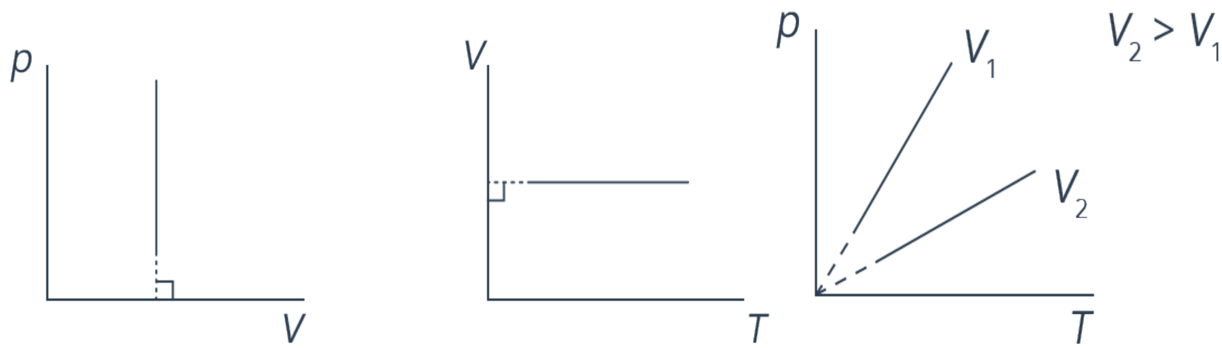


Figure. 9. Graphs of isochors in pV, VT, pT coordinates.

Dalton's law: the pressure of a mixture of ideal gases is equal to the sum of the partial pressures of its components:

$$p = p_1 + p_2 + ... + p_n,$$

where $p_1, p_2, ..., p_n$ are partial pressures. Partial pressure is the pressure of a gas component if it alone occupied the entire volume occupied by the entire mixture.

Normal conditions are defined by the following values:

$$T = 273{,}15\,K \approx 273\,K,$$

$$p = 1{,}0132 \cdot 10^5 \frac{N}{m^2} \approx 101\,kPa,$$

Under normal conditions, one mole of any gas occupies a volume $V = 22{,}415\,m^3 \approx 22\,m^3$.

Problem-solving examples

1. Calculate the amount of substance contained in the gas, if at a pressure of 200 *kPa* and a temperature of 240 *K*, its volume is 40 *l*.

Known quantities:

$p = 200\ kPa = 2 \cdot 10^5\ Pa$

$T = 240\ \mathrm{K}$

$V = 40\ l = 40 \cdot 10^{-3} m^3$

ν - ?

Problem solution

Let's write the Mendeleev Clapeyron equation:

$$pV = \nu RT.$$

The amount of substance is $\nu = \dfrac{pV}{RT}$, $\nu = \dfrac{2 \cdot 10^5 \cdot 40 \cdot 10^{-3}}{8{,}31 \cdot 240} = 4\ mol$.

Answer: 4 mol.

2. During isothermal compression of gas, its volume decreased from 8*l* to 5*l,* and the pressure increased by 60 *kPa*. Determine the initial pressure.

Known quantities:

$V_1 = 8\ l = 8 \cdot 10^{-3} m^3$

$V_2 = 5\ l = 5 \cdot 10^{-3} m^3$

$\Delta p = p_2 - p_1 = 60\ kPa = 60 \cdot 10^3\ Pa$

P_1 - ?

Problem solution

According to the Boyle-Marriott law:

$p_1 V_1 = p_2 V_2$,

$p_1 V_1 = (p_1 + \Delta p) V_2$

$p_1 V_1 = p_1 V_2 + \Delta p V_2$

$$p_1 (V_1 - V_2) = \Delta p V_2,\ p_1 = \frac{\Delta p V_2}{V_1 - V_2},$$

$$p_1 = \frac{60 \cdot 10^3 \cdot V_2}{V_1 - V_2}, \; p_1 = \frac{60 \cdot 10^3 \cdot 5 \cdot 10^{-3}}{8 \cdot 10^{-3} - 5 \cdot 10^{-3}} = 100 \cdot 10^3 \; Pa.$$

Answer: 100 *kPa.*

3. When the absolute temperature increased by 1.4 times, the volume of gas increased by $40 \, sm^3$. Determine the initial volume

Known quantities:	Problem solution
$\frac{T_2}{T_1} = 1.4$	According to Gay-Lussac's law: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$.
$V_2 - V_1 = 40 \, cm^3$	
V_1 - ?	Whence $V_1 = \frac{V_2 \cdot T_1}{T_2}$.

According to the condition of the problem, we can write $V_2 - V_1 = 40 \, cm^3$,

$V_2 = 40 + V_1$.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}, \qquad (1)$$

$$V_2 = 40 + V_1. \qquad (2)$$

(1) and (2) form a system of equations.

$$V_1 = \frac{(40 + V_1)T_1}{T_2} = \frac{40T_1 + V_1T_1}{T_2},$$

$$V_1T_2 = 40T_1 + V_1T_1,$$

$$V_1(T_2 - T_1) = 40T_1, \; V_1(\frac{T_2}{T_1} - 1) = 40.$$

$$V_1 = \frac{40}{\frac{T_2}{T_1} - 1}, \; V_1 = \frac{40}{1{,}4 - 1} = 100 \, cm^3.$$

Answer: $100\,cm^3$.

4. At what temperature was the gas in a closed vessel, if the pressure increased by 1.5 times when it was heated to 140 *K*?

Known quantities:	Problem solution
$\Delta T = 140K$ $\frac{P_2}{P_1} = 1,5$ T_1 - ?	According to Charles' law $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ $\frac{T_2}{T_1} = \frac{p_2}{p_1} = 1,5;$ $T_2 = 1,5T_1$.

Under the condition of the problem, we have: $T_2 - T_1 = 140$

$1,5T_1 - T_1 = 140$, $T_1 = 280\,K$.

Answer: $T_1 = 280\,K$

5. In a container with a capacity of $3l$, there is a gas under a pressure of $2atm$. In a second container with a capacity of $4l$, there is the same gas under a pressure of $1atm$. The temperature in both vessels is the same. Under what pressure is the gas, if you connect both vessels with a tube?

Known quantities:	Problem solution
p -? $V_1 = 3l = 3 \cdot 10^{-3}\,m^3$ $p_1 = 2atm = 2 \cdot 10^5\,Pa$ $V_2 = 4 \cdot 10^{-3}\,m^3$ $p_2 = 1atm = 1 \cdot 10^5\,Pa$ $T_1 = T_2 = T$	According to Dalton's law: $p = p_1' + p_2'$, where p_1' and p_2' are the partial gas pressures after connecting the vessels. According to the Boyle-Mariotta law $p_1'(V_1 + V_2) = p_1V_1;\ p_2'(V_1 + V_2) = p_2V_2$.

where p, V are the pressure and volume; subscripts "1" and "2" refer to the first and second vessels, respectively; dashed values characterize the gas after the compound; not dashed - to the compound. From here:

$$p_1' = \frac{p_1V_1}{V_1+V_2};\ p_2' = \frac{p_2V_2}{V_1+V_2}.$$

Then by Dalton's law:

$$p = \frac{p_1V_1 + p_2V_2}{V_1+V_2}.$$

Substituting numerical data, we get:

$$p = \frac{2\cdot10^5\cdot3\cdot10^{-3} + 10^5\cdot4\cdot10^{-3}}{3\cdot10^{-3}+4\cdot10^{-3}} = 1{,}4\cdot10^5\ Pa$$

Answer: $p = 1{,}4\cdot10^5\ Pa$.

6. The cylinder contains a mass of $80\ g$ oxygen and $320\,g$ argon. The pressure of the mixture is $1 M\Pi a$, the temperature is $300\,K$. Taking these gases as ideal, determine the volume of the cylinder.

Known quantities:

$m_1 = 80\,g = 0{,}08\,kg$

$m_2 = 320\,g = 0{,}320\,kg$

$M_2 = 40\cdot10^{-3}\,kg/mol$

$M_1 = 32\cdot10^{-3}\,kg/mol$

$p = 1MPa = 10^6\,Pa$

$R = 8{,}31 J/mol\cdot K.$

$T = 300\,K$

V -?

Problem solution

According to the equation of state of an ideal gas, the partial pressures of oxygen p_1 and argon p $_2$ are described by formulas :

$$p_1 = \frac{m_1}{M_1}\frac{RT}{V};$$

$$p_2 = \frac{m_2}{M_2}\frac{RT}{V}.$$

According to Dalton's law, the pressure of a mixture of gases is equal to the sum of the partial pressures:

$$p = p_1 + p_2, \quad \text{or} \quad p = \left(\frac{m_1}{M_1} + \frac{m_2}{M_2}\right)\frac{RT}{V}.$$

Whence the volume of the balloon is:

$$V = \left(\frac{m_1}{\mu_1} + \frac{m_2}{\mu_2}\right)\frac{RT}{p}.$$

Let's substitute numerical values and perform calculations:

$$V = \left(\frac{0,08}{32\cdot 10^{-3}} + \frac{0,32}{40\cdot 10^{-3}}\right)\cdot\frac{8,31\cdot 300}{10^6} = 0,0262\ m^3 = 26,2\ l.$$

Answer: $V = 26,2\ l$.

6.3. Properties of steam. Air humidity

Vapour is a gaseous substance that can be liquefied by compression at a given temperature.

A vapour that is in dynamic (moving) equilibrium with its liquid is called a **saturated vapour**.

Properties of saturated vapour: 1) The pressure of saturated vapour remains constant when the vapour volume changes; 2) when the temperature increases, the pressure of saturated vapour increases more rapidly compared to an ideal gas.

Air humidity characterizes the amount of water vapor in the air. Absolute and relative humidity are distinguished.

Absolute humidity is the partial pressure of water vapor (p), present in the air at a given temperature, or the density of water vapor (ρ).

Relative humidity (φ) shows how close water vapor is to saturation.

Relative air humidity is the ratio of the density ρ (pressure p) of water vapor contained in the air at a given temperature to the density ρ_s (pressure p_s) of saturated vapor at the same temperature: it is measured in percent and can be determined by formulas:

$$\varphi = \frac{\rho}{\rho_s} \cdot 100\% = \frac{p}{p_s} \cdot 100\%.$$

Relative humidity is measured in percent.

The most favorable conditions for human activity are at relative air humidity $40-60\%$.

When the air cools, fog appears, dew falls ($\varphi = 100\%$).

The **dew point** is the temperature at which this water vapor becomes saturated.

Problem-solving examples

1. The relative humidity of the air in a room with a volume of $40\,m^3$, at a temperature of $18^0 C$ is equal to 70%. What mass of water should be evaporated in the room so that the water vapour becomes saturated?

Known quantities:

$V = 40\,m^3$

$t = 18^0 C$

$\varphi_1 = 70\% = 0,7$

$\Delta m - ?$

Problem solution

The relative humidity of the air is

$$\varphi = \frac{\rho}{\rho_s} = \frac{m}{V\rho_s},$$

where $\rho_s = 15,4 \cdot 10^{-3} kg/m^3 -$ is the density of saturated water vapour at $18^0 C$, tabular value. From here the mass of water vapour in the room is: $m = \varphi V \rho_s$.

In the initial state $m_1 = \varphi_1 V \rho_s$. In the final state, when water vapour becomes saturated ($\varphi_2 = 100\% = 1$): $m_2 = \varphi_2 V \rho_s$.

The mass of water that was additionally evaporated is equal to:

$$\Delta m = m_2 - m_1 = (\varphi_2 - \varphi_1) V \rho_s .$$

Substituting numerical data, we get:

$$\Delta m = (1 - 0,7) \cdot 40 \cdot 15,4 \cdot 10^{-3} = 184,8 \cdot 10^{-3}\, kg = 0,2\, kg$$

Answer: $\Delta m = 0,2\, kg$

2. A vessel with a volume of $50\, l$ is filled with air at a temperature of $26^0 C$, the relative humidity of which is equal to 80%. The density of saturated water vapor at this temperature is $22,4\ g/m^3$. What will the relative humidity of the air become after some time, if the vessel is filled with water with a mass of $100\, mg$ at the same temperature and maintained it has a constant temperature?

Known quantities:	Problem solution
$V = 50\, l = 50 \cdot 10^{-3} m^3$ $t_1 = 26^0 C$ $\varphi_1 = 80\% = 0,8$ $\rho_s = 22,4\, g/m^3 = 22,4 \cdot 10^{-3} kg/m^3$ $m_2 = 100\, mg = 100 \cdot 10^{-6} kg$ $T = const$ $\varphi_2 - ?$	The relative humidity of the air in the vessel before adding water to the vessel: $\varphi_1 = \dfrac{\rho_1}{\rho_s}$. Let's determine the minimum amount of water that needs to be added to the vessel so that the air becomes saturated ($\varphi = 100\%$) as a result of complete evaporation of this water at $t = const$:

$$\varphi = \frac{\rho}{\rho_s} = \frac{m_1 + m_{\min}}{V} \frac{1}{\rho_s} = \left(\rho_1 + \frac{m_{\min}}{V} \right) \cdot \frac{1}{\rho_s} = \frac{\rho_1}{\rho_s} + \frac{m_{\min}}{V \rho_s} = \varphi_1 + \frac{m_{\min}}{V \rho_s} .$$

From here: $m_{\min} = \left(\varphi - \varphi_1 \right) V \rho_s$,

$$m_{\min} = (\varphi - \varphi_1) V \rho_s = (1 - 0,8) \cdot 50 \cdot 10^{-3} \cdot 22,4 \cdot 10^{-3} = 224 \cdot 10^{-6}\, kg.$$

Since $m_{\min} > m_2$, then the water with a mass of $100\, mg$ will completely evaporate in the vessel.

The relative humidity of the air in the vessel, which will be established after adding water with a mass of m as a result of complete evaporation of this water:

$$\varphi_2 = \frac{\rho_2}{\rho_s} = \frac{m_1 + m}{V} \frac{1}{\rho_s} = \left(\rho_1 + \frac{m}{V} \right) \cdot \frac{1}{\rho_s} = \varphi_1 + \frac{m}{V \rho_s}.$$

Substituting numerical data, we get:

$$\varphi_2 = \varphi_1 + \frac{m}{V \rho_s} = 0,8 + \frac{100 \cdot 10^{-6}}{50 \cdot 10^{-3} \cdot 22,4 \cdot 10^{-3}} = 0,89.$$

Answer: $\varphi_2 = 0,89 = 89\%$.

3. A closed vessel of volume $1\, m^3$, containing $1 kg$ of water, was heated to $150^0 C$. By how much must the volume of the vessel be changed so that the steam in it becomes saturated? The saturated vapor pressure at a temperature of $150^0 C$ is $4,7 \cdot 10^4 Pa$.

Known quantities:

$V_1 = 1 m^3$

$m = 1 kg$

$p_н = 4,7 \cdot 10^4 Pa$

$t = 150^0 C$

$\Delta V - ?$

Problem solution

Gas laws can be applied to vapour. It is necessary to determine the volume of the vessel, which under the given conditions contains only saturated vapour. Let's write the Mendeleev-Clapeyron equation and find the volume of the vessel:

$$pV = \frac{m}{M}RT \Rightarrow V = \frac{m}{M}\frac{RT}{p}, \ T = (273 + t)K$$

Molar mass of water vapour: $M(H_2O) = (2 \cdot 1 + 16) \cdot 10^{-3} = 18 \cdot 10^{-3} kg$.

Substituting numerical data, we get:

$$V_2 = \frac{m}{M}\frac{RT}{p_s} = \frac{1}{18 \cdot 10^{-3}}\frac{8,31 \cdot 423}{4,7 \cdot 10^4} = 4,2 m^3,$$

$$\Delta V = V_2 - V_1 = 4,2 - 1 = 3,2 m^3.$$

Answer: we need to increase the volume of the vessel by $\Delta V = 3,2 m^3$.

6.4. Properties of liquids. Capillary phenomena

Surface tension is a physical phenomenon, the essence of which is the desire of a liquid to reduce its surface area at a constant volume.

The **force of surface tension** is a force acting along the surface of a liquid, perpendicular to the line that bounds the surface and directed in the direction of its contraction $F = \sigma l$ is the surface tension force, where σ is the surface tension coefficient, l is the length of the line bounding the surface.

The SI unit of surface tension is the newton per meter: $[\sigma] = N/m$.

Surface energy is the excess energy of the surface layer of the liquid, caused by the increase in the potential energy of the particles of the surface layer.

Phenomena such as wetting and capillarity are associated with surface tension.

If the interaction of liquid molecules is less than their interaction with the molecules of the contact solid body, then we have a case of **wetting**, and vice versa, when this interaction is greater - **non-wetting** (Fig. 10).

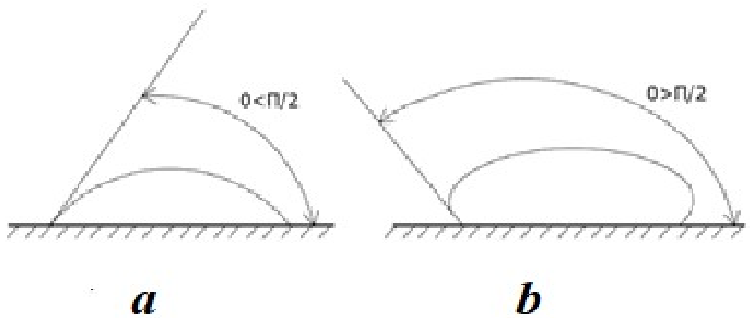


Figure.10. ***a*** is an contact angle $\theta < 90^0$ when wetting, if $\theta = 0^0$ then complete wetting, ***b*** is an contact angle $\theta > 90^0$ without wetting.

Capillarity is the phenomenon of liquid rising (in the case of wetting) or liquid falling (in the case of non-wetting) through thin tubes.

Concave or convex liquid surfaces in thin tubes are called menisci.

The height to which the liquid rises or falls through the capillary is determined by the formula:

$$h = \frac{2\sigma}{\rho g R},$$

where σ is the coefficient of surface tension of the liquid, ρ is the density of the liquid, R is the radius of the capillary.

This formula is also applied to a capillary tube with a non-wetting liquid, only in this case it is not the height of the rise, but the descent of the liquid that is calculated.

Problem-solving examples

1. In a capillary tube with a radius of $0{,}5\,mm$ the liquid rose by $15\,mm$. Determine the density of this liquid. The coefficient of surface tension is $0{,}022\ N/m$. Consider the acceleration of free fall to be $g = 10\,m/s^2$.

Known quantities

$r = 0{,}5\,mm = 0{,}5 \cdot 10^{-3}\,m$

$\sigma = 0{,}022\,N/m$

$g = 10\,m/s^2$

$h = 15\,mm = 15 \cdot 10^{-3}\,m$

$\rho - ?$

Problem solution

In the equilibrium position, the modules of surface tension and gravity are equal $F_t = mg$.

The force of surface tension is $F_t = \sigma l = \sigma \cdot 2\pi r$

The mass of the liquid in the capillary is $m = \rho V = \rho S h = \rho \pi r^2 h$. Then

$$2\pi\sigma r = \rho\pi r^2 hg \Rightarrow \rho = \frac{2\sigma}{rhg}.$$

Substituting numerical data, we get:

$$\rho = \frac{2\sigma}{rhg} = \frac{2 \cdot 0{,}022}{0{,}5 \cdot 10^{-3} \cdot 12 \cdot 10^{-3} \cdot 10} = 7{,}3 \cdot 10^2\,kg/m^3.$$

Answer: $\rho = 7{,}3 \cdot 10^2\,kg/m^3$.

2. Find the mass of water that rose through a capillary tube with a diameter of $0{,}5\,mm$. The coefficient of surface tension is $0{,}022\,N/m$.

Known quantities

$d = 0{,}5\,mm = 0{,}5 \cdot 10^{-3}\,m$

$\rho = 1000\,kg/m^3$

$\sigma = 0{,}022\,N/m$

$m - ?$

Problem solution

If there are no special warnings, we consider wetting to be complete. In the equilibrium position, the modules of surface tension and gravity are equal: $F_t = mg$.

The surface tension is $F_t = \sigma l = \sigma \cdot 2\pi r = \sigma \cdot \pi d$.

Then $\pi\sigma d = mg \Rightarrow m = \dfrac{\pi\sigma d}{g} = \dfrac{3{,}14 \cdot 0{,}022 \cdot 0{,}5 \cdot 10^{-3}}{10} = 3{,}45 \cdot 10^{-6}\,kg$.

<u>Answer:</u> $m = 3{,}45 \cdot 10^{-6}\,kg$

3. The height of water rising in the rice stalk is 15 times higher than in the soil. Determine the diameter of the capillary of the rice if the capillary diameter of the soil is $0{,}3\,mm$.

<u>Known quantities:</u>

$h_1 = 15h_2$

$d_2 = 0{,}3\,mm = 0{,}3 \cdot 10^{-3}\,m$

$d_1 - ?$

<u>Problem solution</u>

The height to which the liquid rises through the capillary is determined by the formula:

$h = \dfrac{2\sigma}{\rho g R}$. The height of the rise of water in the rice stalk is:

$$h_1 = \frac{2\sigma}{\rho g R_1} = \frac{2\sigma}{\rho g \frac{d_1}{2}} = \frac{4\sigma}{\rho g d_1}.$$

The height of water rising in the soil is $h_2 = \dfrac{2\sigma}{\rho g R_2} = \dfrac{4\sigma}{\rho g d_2}$.

According to the conditions of the problem, we have:

$h_1 = 15h_2 \Rightarrow \dfrac{4\sigma}{\rho g d_1} = \dfrac{15 \cdot 4\sigma}{\rho g d_2}$, $d_2 = 15d_1 = 15 \cdot 0{,}3 \cdot 10^{-3} = 4{,}5 \cdot 10^{-3}\,m$.

<u>Answer:</u> $d_2 = 15d_1 = 4{,}5 \cdot 10^{-3}\,m.$

6.5. Problems and tests for self-solving on the topic «Molecular physics»

Problem solving guidelines. The following sequence is recommended when solving problems for calculating gas state parameters. Determine whether the state of the gas changes. If the gas has one state, then the Mendeleev-Clapeyron equation is used. If several gas states are specified, the parameters of these states are recorded. Find out whether the mass of the gas changes. If the mass changes, the Mendeleev-Clapeyron equation is written for each state. Write down additional equations connecting the sought values or parameters of the state. If the problem considers processes related to the change of state of two or more gases, all previous steps must be performed for each gas separately. Solve the resulting system of equations. In gas law problems, you need to use only absolute temperature and immediately convert the temperature value on the Celsius scale to the value on the Kelvin scale. If one of the parameters of the gas remains constant and the mass of the gas does not change, then one of the ideal gas laws is used: Boyle-Mariotte, Gay-Lussac or Charles.

Problems for vapour and humidity in their solution are fundamentally almost no different from problems for ideal gases.

1. Nitrogen with a mass of $7g$ is under a pressure of $0{,}1MPa$ and a temperature of $290K$. As a result of isobaric heating, the nitrogen occupied a volume of $12l$ Determine: 1) the volume of the gas before expansion; 2) gas temperature after expansion; 3) gas densities before and after expansion.

2. A cylinder with a volume of $4l$ contains gas under a pressure of $3atm$, and a second cylinder with a volume of $V_2 = 7l$ contains the same gas under a pressure of $p_2 = 2atm$. What pressure p will be established when the cylinders are connected? The gas temperature is constant.

3. A closed cylindrical vessel with a height of h is divided into two equal parts by a weightless piston that can slide without friction. When the piston is fixed, both halves

are filled with gas, and in one of them the gas pressure is n times greater than in the other. How far will the piston move if it is released? The temperature is considered constant.

4. One cylinder with a volume of $25 cm^3$ contains gas under a pressure of $0,2 MPa$, and in the second - the same gas under a pressure of $1 MPa$. The cylinders, which have the same temperature, are connected by a tube to a faucet. If you open the tap, the pressure in both cylinders is $0,4 MPa$. What is the capacity of the second cylinder?

5. A diatomic gas with a mass of $1 kg$ is under a pressure of $8 \cdot 10^4 Pa$ and has a density of $4 kg / m^3$. Calculate the energy of translational motion of gas molecules under these conditions.

6. Determine the average kinetic energy ε of translational motion of gas molecules under pressure $0,1 Pa$. The concentration of gas molecules is equal to $10^{13} cm^{-3}$.

7. Argon with a mass of $4\ g$ occupies a volume of $0,1\ dm^3$ under a pressure of $3,5\ MPa$. Determine the temperature of the gas, considering the gas to be ideal.

8. Warm air rises. Why is the temperature – 50 °C at an altitude of 10 km?

9. To what temperature must an ideal gas be isobarically heated so that its density is halved compared to the density of this gas when the initial temperature is $0\,^{\circ}C$?

10. Some gas is under a pressure of $700\ \kappa Pa$ at a temperature of $308 K$. Determine its relative molecular weight if the density of the gas is $12,2\ kg / m^3$.

11. Determine the root mean square speed of molecules of an ideal gas, the density of which at normal pressure is equal to $1,2 kg / m^3$.

12. A certain mass of oxygen is at a temperature of $27\,^{\circ}C$ and a pressure of $100\,\kappa Pa$. The kinetic energy of translational motion of oxygen molecules is equal to $6{,}3\,J$. Calculate the number of oxygen molecules.

13. Calculate the kinetic energy of translational motion of two moles of oxygen molecules at a temperature of $17\,^{\circ}C$.

14. A cylinder with a volume of $20\,l$ contains argon under a pressure of $800\,\kappa Pa$ and at a temperature of $325\,K$. After a certain amount of argon was released from the cylinder, the pressure in the cylinder decreased to $600\,\kappa Pa$, and the temperature decreased to $300\,K$. Determine the mass of argon released from the cylinder.

15. Calculate the density of oxygen in the cylinder at a pressure of $1\,MPa$ and a temperature of $300\,K$.

Tests

1. Calculate the mass of a molecule of carbon dioxide. The atomic weights of carbon and oxygen are assumed to be 12 atomic mass unit and 16 atomic mass unit respectively.

A	B	C	D
$7{,}3\cdot10^{-23}\,g$	$7{,}3\cdot10^{-23}\,kg$	$4{,}4\cdot10^{-23}\,g$	$4{,}4\cdot10^{-23}\,kg$

2. What causes gas pressure on the vessel walls?

A	B	C	D
By attracting molecules to each other	The collision of molecules between themselves	Collision of molecules with walls	By attracting molecules to the vessel walls

3. There are approximately $3 \cdot 10^{25}$ gas molecules in the cylinder. What amount of substance is in the cylinder?

A	B	C	D
$0,05 mol$	$0,3 mol$	$300 mol$	$50 mol$

4. Which of the following statements contradicts the basics of the molecular kinetic theory?

A	B	C	D
Matter consists of molecules	Molecules of matter move chaotically	All molecules interact with each other	All molecules of a substance have the same velocity

5. Determine the number of molecules in a volume of $1 cm^3$ ideal gas under normal conditions.

A	B	C	D
$2,65 \cdot 10^{18}$	$2,65 \cdot 10^{19}$	$2,65 \cdot 10^{20}$	$2,65 \cdot 10^{21}$

6. What is the mass of 25 moles of carbon dioxide? The atomic weights of carbon and oxygen are assumed to be 12 atomic mass unit and 16 atomic mass unit respectively.

A	B	C	D
$0,4kg$	$1,1kg$	$40kg$	$110kg$

7. The temperature of which object, from the point of view of physics, makes sense to measure?

A	B	C	D
Electron	Atom	Molecule	Macroscopic body

8. The atoms in the crystal are at such distances from each other that such phenomena are observed

A	B	C	D
The forces of repulsion between them are minimal	The forces of repulsion are equal to the forces of attraction	The forces of attraction between them are minimal	The forces of attraction between them are maximum

9. In liquids, particles oscillate near the equilibrium position, colliding with neighboring particles. From time to time, the particle makes a "jump" to another equilibrium position. What property of liquids can be explained by this nature of particle movement?

A	B	C	D
Little compressibility	Pressure on the bottom of the vessel	Fluidity	Change in volume during heating

10. Determine the number of molecules in a volume of $1cm^3$ ideal gas under normal conditions.

A	B	C	D
$2,65\cdot10^{18}$	$2,65\cdot10^{19}$	$2,65\cdot10^{20}$	$2,65\cdot10^{21}$

11. Molecular oxygen O_2 is in a vessel with a volume of $0,4m^3$ under a pressure of $8,3\cdot10^5 Pa$ and at a temperature of $320K$. What is the mass of oxygen? The molar weight of oxygen is considered equal to 32 atomic mass unit, the universal gas constant is $8,3J/(mol\cdot K)$.

A	B	C	D
$0,2kg$	$2kg$	$0,4kg$	$4kg$

12. What is Brownian motion?

A	B	C	D
Thermal movement of liquid molecules	Ordered movement of liquid molecules	Chaotic movement of particles suspended in a liquid	Orderly movement of particles suspended in a liquid

13. During the isothermal process, the gas pressure increased by 2 times. How did the concentration of gas molecules change?

A	B	C	D
Has doubled	Has grown 4 times	It halved	Has not changed

14. A rigid cylinder at a temperature of $100\,^{0}C$ contains an ideal gas. To what temperature must the cylinder be heated to double the pressure of the gas?

A	B	C	D
$200\,^{0}C$	$373\,^{0}C$	$473\,^{0}C$	$746\,^{0}C$

15. At a constant temperature, the pressure of the gas in the closed cylinder increased by 40%, and the volume decreased by $2l$. Determine the initial volume of the gas.

A	B	C	D
$1l$	$2l$	$3l$	$4l$

7. THERMODYNAMICS

7.1. Internal energy and its change

The **internal energy** of a macroscopic body is equal to the sum of kinetic energies of random motion of all molecules and potential energies of interaction between molecules.

The SI unit of internal energy is the joule, J: $[U]=J$.

Internal energy of a monatomic ideal gas: $U=\frac{3}{2}\nu RT=\frac{3}{2}\cdot\frac{m}{M}RT=\frac{3}{2}pV$, where ν is the amount of substance, m is the mass of the gas, M is the molar mass of the gas, p is the gas pressure, T is the absolute temperature of the gas, V and is the volume.

The change in the internal energy of a monatomic ideal gas is determined by the formula:

$$\Delta U = \frac{3}{2}\nu R\Delta T = \frac{3}{2}\cdot\frac{m}{M}R\Delta T = \frac{3}{2}\Delta\left(pV\right).$$

Internal energy changes during: heat transfer and when performing work on the body or the body itself.

Heat transfer (heat exchange) is the process of changing the body's internal energy without performing work. There are three methods of heat transfer: heat conduction, convection, radiation.

Heat (Q**)** is a physical quantity equal to the energy that a body receives or gives off during heat transfer. The SI unit of heat is the joule, J: $\left[Q\right] = J$.

The **specific heat capacity** of a substance is a physical quantity that is numerically equal to the amount of heat that must be given to a body whose mass is $1kg$, to heat it by $1K$:

$$c = \frac{Q}{m\Delta T}.$$

The SI unit of specific heat capacity is joule per kilogram – Kelvin:

$$\left[c\right] = \frac{J}{kg \cdot K}.$$

When heating or cooling a body whose mass is m, the amount of heat Q is calculated using the formula:

$$Q = cm(T_2 - T_1) = cm\Delta T = cm(t_2 - t_1),$$

where T_1 is the initial body temperature, T_2 is the final temperature, c is the specific heat capacity of the substance.

The specific heat capacity of a substance depends on the type of substance, its aggregate state and the temperature range in which heat transfer occurs.

Thermodynamic work is performed by bodies when their volume changes. During an arbitrary process in an ideal gas, the work is numerically equal to the area of the figure bounded by the process graph in coordinates pV, lines $V = V_1$, $V = V_2$, and axis $p = 0$. During a closed cycle, the work of the gas is numerically equal to the area of the figure bounded by the cycle graph in coordinates pV.

The **work done by an ideal gas** during isoprocesses is calculated by formulas.

Isobaric process: $A = \dfrac{m}{M} R\Delta T = \nu R\Delta T = p\Delta V$,

where $\Delta T = T_2 - T_1$ is the change in temperature, $\Delta V = V_2 - V_1$ is the change in volume;

Isothermal process: $A = \dfrac{m}{M} RT \ln\dfrac{V_2}{V_1} = \dfrac{m}{M} RT \ln\dfrac{p_1}{p_2}$;

Isochoric process: $A = 0$.

Problem-solving examples

1. During isobaric heating to a temperature of $600K$ the gas done the work equal to $8,31kJ$. Determine the initial absolute temperature of nitrogen with a mass of $0,56kg$. The molar mass of nitrogen is $0,028kg/mol$, the universal gas constant is $R = 8,31J/(mol \cdot K)$.

Known quantities:

$p = const$

$T_2 = 600K$

$A = 8,31kJ = 8,31 \cdot 10^3 J$

$m = 0,56kg$

$M(N_2) = 28 \cdot 10^{-3} kg / mol$

$T_1 - ?$

Problem solution

The work of an ideal gas during an isobaric process is calculated by the formula:

$$A = \frac{m}{M} R \Delta T = \frac{m}{M} R\left(T_2 - T_1\right),$$

where $R = 8,31\ J / (mol \cdot K)$, from here:

$$T_2 - T_1 = \frac{AM}{mR} \Rightarrow T_1 = T_2 - \frac{AM}{mR}.$$

Substituting numerical data, we get:

$$T_1 = T_2 - \frac{AM}{mR} = 600 - \frac{8,31 \cdot 10^3 \cdot 28 \cdot 10^{-3}}{0,56 \cdot 8,31} = 550\,K.$$

Answer: $T_1 = 550\,K$.

2. A monatomic gas, the mass of which is $1kg$, is under a pressure of $80kPa$, its density is $4kg / m^3$. Determine the internal energy of the gas.

Known quantities

$m = 1kg$

$p = 80\,kPa$

$\rho = 4kg / m^3$

$U - ?$

Problem solution

The internal energy of a monatomic ideal gas is: $U = \frac{3}{2} pV$,

where p is the gas pressure, V is the volume of the gas.

The volume of gas can be determined from the formula for the mass of gas:

$m = \rho V \Rightarrow V = \dfrac{m}{\rho} \Rightarrow U = \dfrac{3}{2} pV = \dfrac{3}{2} p \cdot \dfrac{m}{\rho}$. Substituting numerical data, we get:

$$U = \frac{3}{2}\frac{pm}{\rho} = \frac{3}{2} \cdot \frac{80 \cdot 10^3 \cdot 1}{4} = 30 \cdot 10^3\, J = 30\, kJ.$$

Answer: $U = 30\, kJ$.

3. As a result of the isothermal expansion of nitrogen at a temperature of $7\,^\circ C$ its volume doubled. Determine the work done during the expansion of the gas. The mass of nitrogen is $200\, g$. The molar mass of nitrogen is $M(N_2) = 28 \cdot 10^{-3}\, kg/mol$.

Known quantities:

$t = 7\,^\circ C$

$V_2 = 2V_1$

$M(N_2) = 28 \cdot 10^{-3}\, kg/mol$

$m = 200\, g = 0,2\, kg$

$A - ?$

Problem solution

The work done by an ideal gas during an isothermal process is $(T = const)$:

$$A = \frac{m}{M} RT \ln \frac{V_2}{V_1} = \frac{m}{M} RT \ln \frac{2V_1}{V_1} = \frac{m}{M} RT \ln 2,$$

where $R = 8,31 \dfrac{J}{mol \cdot K}$, $T = (273 + t\,^\circ C)K \Rightarrow$

$$A = \frac{m}{M} RT \ln 2 = \frac{0,2}{28 \cdot 10^{-3}} 8,31 \cdot 300 \cdot \ln 2 = 11 J$$

Answer: $U = 11 J$.

7.2. Change of aggregate states of matter. Heat balance equation

Melting is the transition of a substance from a solid crystalline state to a liquid state. The reverse process is called crystallization. The amount of heat that must be spent on melting a certain mass of substance is equal to the amount of heat released during the crystallization of this substance.

The temperature of the substance during melting and crystallization does not change. The melting and crystallization temperatures for the same substance are the same.

The **amount of heat required to melt a crystalline substance** at the melting temperature is calculated using the formula:

$$Q = \lambda m,$$

where Q is the amount of heat absorbed by a solid crystalline substance; λ is the specific heat of melting (crystallization) m is the mass of the substance, kg.

The SI unit of specific heat of fusion is joule per kilogram: $[\lambda] = \frac{J}{kg}$.

Specific heat of melting (λ**)** is a physical quantity that is numerically equal to the amount of heat required to transform 1 kg of a crystalline substance taken at its melting point into a liquid.

Data on the specific heat of fusion of various substances are contained in the table.

Evaporation is the transition of a substance from a liquid to a gaseous state. The reverse process is called condensation. There are two types of vaporization - evaporation and boiling.

In order for vaporization to occur at a constant temperature (boiling temperature) to a liquid whose mass is m, it is necessary to provide the amount of heat Q, calculated by the formula:

$$Q = Lm,$$

where L is the specific heat of vaporization (condensation). The same amount of heat is released during steam condensation.

The SI unit of specific heat of vaporization is the joule per kilogram $[L] = \frac{J}{kg}$.

The **specific heat of vaporization (condensation)** L is a physical quantity that is numerically equal to the amount of heat required to convert 1 kg of liquid taken at the boiling temperature into vapour.

Data on the specific heat of vaporization (condensation) of various liquids are contained in the table.

The process of burning fuel is accompanied by the release of energy.

The **amount of heat** Q**, released during fuel combustion** is calculated using the formula:

$$Q = qm,$$

where q is the specific heat of fuel combustion, which depends on the type of fuel.

The SI unit of the specific heat of combustion of fuel is the joule per kilogram:

$$[q] = \frac{J}{kg}.$$

Data on the specific heat of combustion of various types of fuel are contained in the table.

The **specific heat of fuel combustion** is numerically equal to the amount of heat released during the complete combustion of 1 kg of fuel.

The **coefficient of useful efficiency of the heater** is a physical value that characterizes the efficiency of the heater and is equal to the ratio of usefully consumed heat to all the heat that can be released during complete combustion of fuel:

$$\eta = \frac{Q_{usef}}{Q_{all}} = \frac{Q_{usef}}{qm}.$$

Heat balance equation: In an isolated system of bodies, in which the internal energy of the bodies changes only due to heat transfer, the total amount of heat given by some bodies of the system is equal to the total amount of heat received by other bodies of this system:

$$Q_1^- + Q_2^- + ... + Q_n^- = Q_1^+ + Q_2^+ + ... + Q_n^+$$

The heat balance equation describes the law of conservation of energy during heat exchange.

Problem-solving examples

1. At what speed will a lead ball melt upon hitting a partition? The initial temperature of the ball is $30^0 C$. Assume that upon impact, 50% of the mechanical energy was converted into internal energy. The melting point of lead is $330^0 C$. The specific heat of lead $c = 120 J/(kg \cdot^0 C)$, the specific heat of fusion of lead is $\lambda = 2,5 \cdot 10^4 J/kg$.

Known quantities:

$t_1 = 30^0 C$

$t_m = 330^0 C$

$\eta = 50\% = 0,5$

$c = 130 J/(kg \cdot^0 C)$

$\lambda = 2,5 \cdot 10^4 J/kg$

υ_1 -?

Problem solution

The change in the internal energy of the ball occurs due to the implementation of mechanical work against resistance forces, i.e. $\Delta U = -A$.

The work is determined from the kinetic energy theorem:

$A = \Delta E_k = E_{k2} - E_{k1} = \frac{m\upsilon_2^2}{2} - \frac{m\upsilon_1^2}{2}$, where m is the mass of the bullet, υ_1 is the initial velocity, υ_2 is the final velocity of the ball, which is equal to zero under the condition of the problem, then $A = -\frac{m\upsilon_1^2}{2}$.

According to the condition of the problem, 50% of the mechanical (kinetic) energy goes to change the internal energy of the ball, that is, to heating and melting lead:

$$\eta \Delta E_k = \Delta U\,,\;\; \Delta U = Q_h + Q_m\,,$$

$$Q_h = cm(t_m - t_1)\,,\; Q_m = \lambda m\,.$$

Hence

$$\eta \cdot \frac{m\upsilon_1^2}{2} = cm\left(t_m - t_1\right) + \lambda m\,,\; \upsilon_1 = \sqrt{\frac{2(c(t_m - t_1) + \lambda)}{\eta}}\,.$$

Substituting numerical data, we get:

$$\upsilon_1 = \sqrt{\frac{2(c(t_m - t_1) + \lambda)}{\eta}} = 506\, m/s$$

Answer: $\upsilon_1 = 506\, m/s$.

2. There is wet snow with a mass of $1{,}5\, kg$ in a calorimeter with a heat capacity of $1{,}18 \cdot 10^3\, J/K$. Vapour with a mass of $100\, g$ is injected into the calorimeter, after which the temperature is set to $283\, K$. Determine the mass of water in the snow.

Known quantities:

$m_1 = 100\,g = 0,1kg$

$m = 1,5\,кг$

$T = 283K$

$T_1 = 373K$

$T_2 = 273K$

$C = 1,18 \cdot 10^3 J / K$

$L = 22,6 \cdot 10^5 J / kg$

$\lambda = 3,36 \cdot 10^5 J / kg$

$c = 4200 J / kg \cdot K$

$m_w - ?$

Problem solution

According to the heat balance equation: $Q_{lost} = Q_{gained}$.

Vapour gives off heat during condensation $Q_1 = Lm_1$.

After condensation of vapour water is formed at a temperature of $T_1 = 373\,K$.

Water from vapour (the mass of water from vapour is equal to the mass vapour) cools to the equilibrium temperature $Q_2 = cm_1(T_1 - T)$.

Then $Q_{lost} = Q_1 + Q_2 = Lm_1 + cm_1(T_1 - T)$.

Wet snow (m) is a mixture of water (Δm) and ice ($m_2 = m - \Delta m$).

The heat released by the steam is used to melt ice from snow $Q_3 = \lambda(m - \Delta m)$, heat water from snow $Q_4 = cm(T - T_2)$ and heat the calorimeter $Q_5 = C(T - T_2)$.

$Q_{gained} = Q_3 + Q_4 + Q_5$,

$Q_{gained} = Q_3 + Q_4 + Q_5 = \lambda(m - \Delta m) + cm(T - T_2) + C(T - T_2)$.

The heat balance equation will have the form:

$$Lm_1 + cm_1(T_1 - T) = \lambda(m - \Delta m) + cm(T - T_2) + C(T - T_2)$$

From here we determine the mass:

$$\Delta m = \frac{\lambda m - Lm_1 - cm_1(T_1 - T) + c(T - T_2) + C(T - T_2)}{\lambda}.$$

Substituting numerical data, we get $\Delta m = 0,6\,kg$.

Answer: $\Delta m = 0,6\,kg$

3. How much wood must be burned in a furnace with an efficiency of 40%, if it is necessary to heat water to $30\,K$ with a volume of $5\,l$ in an aluminum calorimeter with a mass of $3\,kg$. The heat of firewood combustion is $q = 10\frac{MJ}{kg}$. The specific heat capacity of water is $c_w = 4200\frac{J}{kg \cdot K}$. The specific heat capacity of aluminum is $c_{AL} = 880\frac{J}{kg \cdot K}$.

Known quantities:

$\eta = 40\% = 0,4$

$V = 5l = 5 \cdot 10^{-3}\,m^3$

$\rho = 1000\frac{kg}{m^3}$

$m_{AL} = 3\,kg$

$\Delta T = 30\,K$

$q = 10\frac{MJ}{kg}$

$c_w = 4200\frac{J}{kg \cdot K}$

$c_{AL} = 880\frac{J}{kg \cdot K}$

$m - ?$

Problem solution

The process of burning fuel is accompanied by the release of energy. Only part of the amount of heat Q_1, released during the burning of firewood is used to heat water Q_2 and the calorimeter Q_3:

$$\eta = \frac{Q_2 + Q_3}{Q_1}.$$

The amount of heat Q_1, released during firewood combustion: $Q_1 = qm$.

The amount of heat needed to heat water:

$$Q_2 = c_w m_w \Delta T = c_w \rho V \Delta T.$$

The amount of heat required to heat the calorimeter: $Q_3 = c_{AL} m_{AL} \Delta T$.

Then

$$m = \frac{c_w \rho V \Delta T + c_{AL} m_{AL} \Delta T}{\eta q}.$$

Substituting numerical data, we get:

$$m = \frac{4200 \cdot 1000 \cdot 5 \cdot 10^{-3} \cdot 30 + 880 \cdot 3 \cdot 30}{0,4 \cdot 10 \cdot 10^6} = 0,177\,kg.$$

<u>Answer:</u> $m = 0,177\,kg$

7.3. First law of thermodynamics

The **first law of thermodynamics**: the amount of heat Q, transferred to the system is used to change its internal energy ΔU and to perform work by the system A: $Q = \Delta U + A$.

The change in the internal energy of a monatomic ideal gas is calculated using formulas $\Delta U = \frac{3}{2}\frac{m}{M} R \Delta T = \frac{3}{2} \nu R \Delta T = \frac{3}{2} p \Delta V$.

For an isothermal process, equation $T = const \Rightarrow \Delta T = 0,$ is fulfilled then $\Delta U = 0$ and the first law of thermodynamics has the form $Q = A$.

The gas work during an isothermal process is $A = \frac{m}{M} RT \ln \frac{V_2}{V_1} = \frac{m}{M} RT \ln \frac{p_1}{p_2}$.

For an isochoric process, equation $V = const \Rightarrow \Delta V = 0,$ is fulfilled then $A = 0$ and the first law of thermodynamics has the form: $Q = \Delta U$.

For an isobaric process, equation: $A = \frac{m}{M} R\Delta T = p\Delta V$, and the first law of thermodynamics has the form: $Q = \Delta U + p\Delta V$ or $Q = \Delta U + \frac{m}{M} R\Delta T$.

An **adiabatic process** is a process that occurs without heat exchange with the environment $Q = 0$. The first law of thermodynamics for an adiabatic process has the form: $\Delta U = -A$, that is, the system performs work by reducing its internal energy.

Problem-solving examples

1. 1 kmol of monatomic gas heats up by 80% under conditions of free expansion. Find: 1) the amount of heat transferred to the gas, 2) the change in its internal energy, 3) the work of expansion.

Known quantities:

$\nu = 1 kmol = 10^3 mol$

$\Delta t = \Delta T = 80^0 C = 80 K$

$Q, \Delta U, A$ -?

Problem solution

The work of gas expansion under the conditions of free expansion, that is, during an isobaric process, is:

$$A = \frac{m}{M} R\Delta T = p\Delta V = \nu R\Delta T = 10^3 \cdot 8{,}31 \cdot 80 = 6{,}64 \cdot 10^5 J, \text{ so}$$

$$A = \nu R\Delta T = 10^3 \cdot 8{,}31 \cdot 80 = 6{,}64 \cdot 10^5 J$$

where m, M is the mass and molar mass of air; p, V, T are the pressure, volume and absolute temperature of the air; R is a universal gas constant; ν is the number of moles of the substance. The change in internal energy is equal to:

$$\Delta U = \frac{3}{2}\nu R\Delta T = \frac{3}{2} A = \frac{3}{2} \cdot 6{,}64 \cdot 10^5 = 9{,}96 \cdot 10^5 J.$$

According to the first law of thermodynamics:

$Q = \Delta U + A = (6,64 + 9,96) \cdot 10^5 = 16,60 \cdot 10^5 J$.

Answer: $A = 6,64 \cdot 10^5 J$; $\Delta U = 9,96 \cdot 10^5 J$; $Q = 16,60 \cdot 10^5 J$.

2. What work is done by 2 moles of gas at isobaric heating to 50 K? How much heat did the gas get?

Known quantities:
$\nu = 2 mol$
$\Delta T = 50K$
$A - ?,\ Q - ?$

Problem solution

For an isobaric process, we can write $A = p\Delta V = \nu R \Delta T$, and substituting numerical data, we get:

$$A = \nu R \Delta T = 2 \cdot 8,31 \cdot 50 = 831 J.$$

The change in internal energy is:

$$\Delta U = \frac{3}{2}\nu R \Delta T = \frac{3}{2} A.$$

The first law of thermodynamics for an isobaric process has the form:

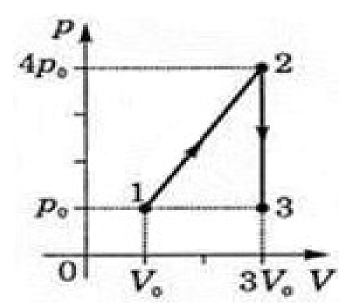
$$Q = \Delta U + A = \frac{3}{2} A + A = \frac{5}{2} A.$$

Substituting numerical data, we get: $Q = \frac{5}{2} A = \frac{5}{2} \cdot 831 = 2077,5 J$.

Answer: $A = 831 J$; $Q = 2077,5 J$.

3. With an ideal monatomic gas with a constant mass, the processes 1-2-3 occur, shown in the graph (see figure). How much heat did the gas receive in processes 1-2-3, if $p_0 = 101 kPa$, $V_0 = 12 l$?

Known quantities:
$p_0 = 101kPa = 101 \cdot 10^3 Pa$
$V = 12l = 12 \cdot 10^{-3} m^3$
Q-?



Problem solution

According to the first law of thermodynamics $Q = \Delta U + A$.

The equation of state of an ideal gas (Mendeleev-Clapeyron equation) can be formulated as follows:

$$pV = \frac{m}{M}RT.$$

Section 1-2-3:

$$p_1 = p_0,$$
$$V_1 = V_0,$$
$$p_2 = 4p_0,$$
$$V_2 = 3V_0,$$
$$p_3 = p_0,$$
$$V_3 = 3V_0,$$

$$\Delta U = \frac{3}{2}\nu R\Delta T = \frac{3}{2}(p_2V_2 - p_1V_1) = \frac{3}{2}(4p_0 \cdot 3V_0 - p_0V_0) = 16{,}5\,p_0V_0.$$

The gas work is numerically equal to the area under the graph of the function: $p = p(V)$

$$A_{1-2} = p_0(3V_0 - V_0) + \frac{1}{2}(4p_0 - p_0)(3V_0 - V_0) = 5p_0V_0.$$

Section 2-3:

$$\Delta U = \frac{3}{2}\nu R(T_3 - T_2) = \frac{3}{2}(p_3V_3 - p_2V_2) =$$
$$= \frac{3}{2}(p_0 \cdot 3V_0 - 4p_0 \cdot 3V_0) = -13{,}5\,p_0V_0.$$

Work done by gas on section 2-3:

$$A_{2-3} = p\Delta V_{2-3} = 0.$$

The amount of heat received by the gas during the process:

$$Q = (\Delta U_{1-2} + A_{1-2}) + (\Delta U_{2-3} + A_{2-3}) =$$
$$= 16,5\, p_0 V_0 + 5\, p_0 V_0 - 13,5\, p_0 V_0 + 0 = 8\, p_0 V_0.$$

Substituting numerical data, we get:

$$Q = 8 p_0 V_0 = 8 \cdot 101 \cdot 10^3 \cdot 12 \cdot 10^{-3} = 9696\, J = 9,7\, kJ.$$

Answer: $Q = 9,7\, kJ$.

7.4. Carnot cycle. The efficiency coefficient of the heat engine

The efficiency of the heat engine is equal to:

$$\eta = \frac{A}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1},$$

where A is the work done by the body per cycle, Q_1 is the amount of heat received by the body from the heater, Q_2 is the amount of heat that the body gives to the refrigerator.

For an ideal Carnot cycle, the efficiency is:

$$\eta = \frac{T_1 - T_2}{T_1},$$

where T_1 is the absolute temperature of the heater, T_2 is the absolute temperature of the refrigerator.

Problem-solving examples

1. The heater of a heat engine operating according to the Carnot cycle has a temperature of $473\, K$. What is the temperature of the refrigerator, if for each kilojoule of heat received from the heater, the machine does work that is equal to $0,4\, kJ$?

Known quantities:

$Q = 1kJ = 10^3 J$

$T_1 = 473K$

$A = 0,4kJ = 400J$

$T_2 - ?$

Problem solution

The temperature of the refrigerator can be found by using the expression for the efficiency of a machine operating on the Carnot cycle:

$\eta = \frac{T_1 - T_2}{T_1}$, where T_1 is the absolute temperature of the heater, T_2 is the absolute temperature of the refrigerator. From here: $T_2 = T_1(1-\eta)$.

The efficiency of the machine is equal to $\eta = \frac{A}{Q}$, where A is the useful work done by the machine; Q is the heat that is supplied to the machine in the same time.

$$T_2 = T_1(1-\eta) = T_1(1-\frac{A}{Q}) = 473 \cdot (1-\frac{400}{10^3}) = 283,8K.$$

Answer: $T_2 = 283,8K$.

2. Determine the temperature of the heater if an ideal heat engine operating according to the Carnot cycle gives 75% of the heat received from the heater to the refrigerator. The temperature of the refrigerator is $300K$.

Known quantities:

$Q_2 = 0,75Q_1$

$T_2 = 300K$

$T_1 - ?$

Problem solution

The efficiency of the heat engine is $\eta = \frac{Q_1 - Q_2}{Q_1}$; For an ideal Carnot cycle, the efficiency is: $\eta = \frac{T_1 - T_2}{T_1}$.

We equate both values of efficiency $\frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$, or $1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$.

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}; \Rightarrow T_1 = \frac{Q_1 \cdot T_2}{Q_2}; \; T_1 = \frac{Q_1 \cdot 300}{0,75 Q_1} = 400\,K.$$

Answer: $T_1 = 400\,K$.

3. An ideal refrigerating machine, which works according to the reverse Carnot cycle, does $37\,kJ$ work in one cycle. At the same time, heat is taken from a body with a temperature of $-10^o C$ and transferred to a body with a temperature of $17^o C$. Determine: 1) efficiency of the cycle; 2) cooling coefficient; 2) the amount of heat that is removed from a cold body in one cycle; 3) the amount of heat that is given to a hot body in one cycle.

Known quantities:

$A = 37\,kJ = 37000\,J$

$t_2^o = -10^o C$

$T_2 = 263\,K$

$t_1^o = 17^o C$

$T_1 = 290\,K$

$\eta - ?, \eta_x - ?, Q_2 - ?, Q_1 - ?$

Problem solution

The temperature must be converted from the Celsius scale to the Kelvin scale. We find the efficiency using the formula

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{290 - 263}{290} = 0,0931.$$

We can determine the cooling coefficient using the formula:

$$\eta_x = \frac{T_2}{T_1 - T_2} = \frac{263}{190 - 263} = 9,74.$$

The heat taken from a cold body is equal:

$$Q_2 = \eta_x A = 9,74 \cdot 37000 = 360\,kJ.$$

The heat Q_1, given to a hot body can be found in two ways:

by formula $Q_1 = A + Q_2 = 37000 + 360000 = 397\ kJ$,

or by formula $Q_1 = \dfrac{A}{\eta} = \dfrac{37}{0,0931} = 397\ kJ$.

<u>Answer:</u> $\eta = 0,0931$, $\eta_x = 9,74$, $Q_2 = 360\,kJ$, $Q_1 = 397\,kJ$.

7.5. Problems and tests for self-solving on the topic «The foundations of thermodynamics»

Problem solving guidelines. When solving tasks on thermal phenomena, it is necessary to establish which bodies are included in this thermodynamic system, as well as to find out what is the cause of the change in the internal energy of the bodies of the system. In the case of an adiabatically isolated closed system, it should be established in which bodies of the system the internal energy increases and in which it decreases. Find out whether phase transitions (evaporation or condensation, melting or crystallization) occur in the system of bodies. At the same time, it is necessary to use the graph of the dependence of the temperature change of bodies on the amount of heat received or given during heat exchange. Compile the heat balance equation, while remembering that in this sum the terms corresponding to the heat of fusion of solids or the heat of vaporization of liquids are included with a "+" sign, and the terms corresponding to the heat of crystallization of solid bodies or the heat of condensation of vapour with sign "–".

When considering processes in which heat exchange occurs with the environment and mechanical work is carried out, the first law of thermodynamics is written in the form: $Q = \Delta U + A$, where Q is the amount of heat transferred to the system, ΔU is the change in the internal energy of the system, A is the work done by the system.

1. $500\ J$ of heat was transferred to the diatomic gas. At the same time, the gas expands at $p = const$. Calculate the work and change in internal energy.

2. During isobaric compression at an initial temperature of $100\ ^\circ C$ the volume of oxygen decreased by $1{,}25$ times. The mass of oxygen is equal to $10\ kg$. Determine the work of compression and the amount of removed heat.

3. A vapour engine with a power of $14{,}7\ kW$ consumes a mass of $8{,}1\ kg$ coal with a calorific value of $3{,}3{\cdot}10^7\ J/kg$ in one hour of operation. The heater temperature is $200^0 C$, the refrigerator temperature is $60\ ^0C$. Determine the actual efficiency of the machine η_1 and compare it with the efficiency η_2 of a heat engine operating on the Carnot cycle between the same temperatures.

4. An ideal gas performing the Carnot cycle gives 2/3 of the heat received from the heater to the refrigerator. The temperature of the refrigerator is $280\ K$. Determine the temperature of the heater.

5. A piece of copper with a mass of $300\ g$ at a temperature of $97\ ^\circ C$ was placed in a calorimeter containing water with a mass of $100\ g$ at a temperature of $7\ ^\circ C$. Determine the increase in entropy of the system until the temperature equalizes.

6. The process of expansion of 3 moles of argon occurs so that the pressure of the gas increases in direct proportion to its volume. Find the increase in gas entropy when its volume doubles.

7. Determine the internal pressure of one kilomol of nitrogen under normal conditions.

8. Calculate the internal energy of carbon dioxide with a mass of 132 g under normal conditions, when the gas is considered ideal.

9. What amount of heat is needed to heat air with a mass of $77,4 kg$ from 10 to $20^0 C$?

10. When cooling copper with a mass of $100\ g$ to a temperature of $22\ ^{\circ}C$, $4\ kJ$ heat was released. Determine the initial temperature of copper.

11. Water with a mass of $200 kg$ was poured into a steel tank with a mass of $65 kg$. As a result of heating, the temperature of the water rose from $4\ ^{\circ}C$ to $29\ ^{\circ}C$. How much heat did the tank and water receive?

12. A copper bar with a mass of $1 kg$ at a temperature of $90\ ^{\circ}C$ is lowered into water with a mass of $300\ g$ at a temperature of $15\ ^{\circ}C$. What temperature will it reach?

13. A diatomic gas, the mass of which is $1 kg$, is under a pressure of $80 kPa$. The density of the gas is $4 kg / m^3$. Determine the internal energy of the gas.

14. As a result of isothermal expansion in the Carnot cycle, the gas received $150 kJ$ heat under the heater. Determine the work of isothermal compression of this gas, when it is known that the efficiency of the cycle is $\eta = 0,4$.

15. The gas performs the Carnot cycle. The work of isothermal expansion of gas is equal to $5 J$. Determine the work of isothermal compression, if the thermal efficiency of the cycle is 0,2.

Tests

1. In a heat engine, the gas at some stage of the cycle received heat equal to $300 J$ and performed work equal to $60 J$. How did the internal energy of the gas change at this stage of the cycle?

A	B	C	D
Increased on $240 J$	Increased on $360 J$	Decreased on $60 J$	Decreased on $240 J$

2. The Carnot cycle consists of

A	B	C	D
two isotherms and two isochores	two isochores and two isobars	two isotherms and two adiabats	two isobars and two adiabats

3. Hot vapour enters the turbine at a temperature of $500\,^0C$, and leaves it at a temperature of $130\,^0C$. What is the efficiency of the turbine? A vapour turbine can be considered an ideal heat engine.

A	B	C	D
26%	92%	74%	100%

4. How does the internal energy of the air in the room change when it is heated at constant pressure?

A	B	C	D
It is decreasing	It is increasing	Does not change	The answer depends on the mass of the gas

5. Under what conditions does water evaporate?

A	B	C	D
Only at a temperature of $100\,^{0}C$	At a temperature above $100\,^{0}C$	At a temperature above the critical temperature	At any water temperature

6. The heat engine works according to the Carnot cycle. The temperature of the heater is $100\,^{0}C$, and the temperature of the refrigerator is $0\,^{0}C$. During time $1s$ the working body of the engine gives the refrigerator heat equal to $27{,}3kJ$. What amount of heat does the working body receive from the heater during time $1s$?

A	B	C	D
$37{,}3kJ$	$37{,}3J$	$373kJ$	$373J$

7. Helium under a pressure of 2 atm and occupying a volume of 3 *l* has the same internal energy as neon occupying a volume of 5 *l*. Determine the pressure of neon.

A	B	C	D
$5/6$ atm	$1{,}2$ atm	$10/3$ atm	1 atm

8. The absolute temperature of the refrigerator of an ideal heat engine is 3 times less than the temperature of the heater. If the temperature of the heater is reduced by 2 times, and the temperature of the refrigerator does not change, then the efficiency of the heat engine will decrease by

A	B	C	D
1,5 times	2 times	3 times	4,3 times

9. When the volume of a monatomic gas decreased by 4 times, its pressure increased by 20%. How many times did the internal energy of the gas change?

A	B	C	D
$\frac{U_2}{U_1}=4,8$	$\frac{U_2}{U_1}=1,2$	$\frac{U_2}{U_1}=0,3$	$\frac{U_2}{U_1}=4$

10. How much heat did the gas receive or give up, if the work done by it is $160J$, and its internal energy decreased by $90J$?

A	B	C	D
Received $70J$	Received $250J$	Gave $70J$	Gave $250J$

11. In the process of adiabatic expansion, the gas performs work equal to $3\cdot10^{10}J$. What is the change in the internal energy of the gas?

A	B	C	D
0	$3\cdot10^{10}J$	$-3\cdot10^{10}J$	Any numerical value

12. During crystallization, the kinetic energy of the particles of the substance …

A	B	C	D
Does not change	Decreases	Increases	At first it decreases, then does not change

14. During boiling, the kinetic energy of particles of liquid …

A	B	C	D
Does not change	Decreases	Increases	At first it decreases, then does not change

15. After isochoric heating to $400K$ the internal energy of air with a mass of $150\,g$ has increased by $14{,}7\,kJ$. Determine the initial temperature of the air, assuming that its heat capacity in this case is $700J/(kg \cdot K)$.

A	B	C	D
$200K$	$260K$	$300K$	$320K$

APPENDICES

Table 1. Fundamental constants

Speed of light in vacuum	$c = 2,998 \cdot 10^{8} m / s$
Gravitational constant	$G = 6,6742 \cdot 10^{-11} N \cdot m^{2} / kg^{2}$
Gas constant	$R = 8,3144 J / mol \cdot K$
Avogadro constant	$N_A = 6,022 \cdot 10^{23} mol^{-1}$
Boltzmann constant	$k = 1,38 \cdot 10^{-23} J / K$
Elementary charge	$e = 1,6 \cdot 10^{-19} C$
Electron mass	$m_e = 9,11 \cdot 10^{-31} kg$
Proton mass	$m_p = 1,6726 \cdot 10^{-27} kg$
Neutron mass	$m_n = 1,6749 \cdot 10^{-27} kg$

Table 2. Properties of some liquids at 20^{0}C

Liquid	Density, kg/m^3	Boiling point, $^\circ C$	Specific heat of vaporization, kJ/kg	Specific heat capacity, J/(kg·K)
Water	1000	100	2300	4200
Ethanol	790	850	850	2400

Table 3. Properties of some solids

Substance	Density, kg/m^3	Boiling point, K	Specific heat capacity $J/(kg \cdot K)$	Specific heat of fusion, J/kg
Aluminum	$2{,}7 \cdot 10^3$	932	$9{,}2 \cdot 10^2$	$3{,}8 \cdot 10^5$
Iron	$7{,}8 \cdot 10^3$	1803	$4{,}6 \cdot 10^2$	$2{,}7 \cdot 10^5$
Zinc	$7{,}1 \cdot 10^3$	692	$4{,}0 \cdot 10^2$	$1{,}18 \cdot 10^5$
Copper	$8{,}9 \cdot 10^3$	1356	$3{,}8 \cdot 10^2$	$1{,}8 \cdot 10^5$
Tin	$7{,}3 \cdot 10^3$	505	$2{,}5 \cdot 10^2$	$5{,}8 \cdot 10^4$
Lead	$1{,}14 \cdot 10^4$	600	$1{,}2 \cdot 10^2$	$2{,}5 \cdot 10^4$
Silver	$1{,}05 \cdot 10^4$	1233	$2{,}5 \cdot 10^2$	$8{,}8 \cdot 10^4$
Ice	$0{,}9 \cdot 10^3$	273	$2{,}09 \cdot 10^3$	$3{,}35 \cdot 10^5$

Table 4. Some prefixes for powers of ten

Prefix	Abbreviation	Power	Prefix	Abbreviation	Power
tera	T	10^{12}	milli	m	10^{-3}
giga	G	10^9	micro	μ	10^{-6}
mega	M	10^6	nano	n	10^{-9}
kilo	k	10^3	pico	p	10^{-12}

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