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NUMERICAL STUDY OF THE INFLUENCE OF THE LENGTH OF A ROD ON ITS CRITICAL FORCES

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Abstract: The effect of changing the length of a longitudinally compressed rod on its critical forces is numerically investigated. The research is carried out on the example of a two-span rectilinear rod of bending stiffness constant along length, which is compressed by a constant lengthwise longitudinal force and hinged on one of the ends on an absolutely rigid support, and inside - on a support of finite stiffness. The change in the length of the rod occurs due to the movement of the end hinge support with the corresponding increase or decrease of the adjacent section of the rod without changing the position and characteristics of the remaining constraints. The dependence of the critical forces of the rod on the position of this support and, accordingly, on the length of the adjacent compressed section of the rod is investigated. Calculations are performed on the basis of the use of known exact analytical expressions of the influence functions of a rod of constant cross-section compressed by a longitudinal force constant by length. In the considered examples, qualitative signs of increase, decrease, and extremum of simple critical forces when changing the length of the rod, related to the qualitative features of the corresponding buckling forms, established earlier theoretically, were fully confirmed. In particular, exact calculations have shown that the increase or decrease of the simple critical force when the length of the fragment of the rod adjacent to the movable support is changed is determined by the type of the corresponding buckling form in the neighborhood of this support. Different possible configurations of buckling forms are considered, and the behavior of critical forces when changing the length of the rod are considered for each of the configurations. In order to verify the previously established theoretical results, which relate to the study of the behavior of not only the main critical forces, but also higher simple critical forces, which have an arbitrary number in the spectrum, the calculations are carried out in the article for the second critical forces of the rods considered in the given examples. The results of the calculations are shown in the form of graphs, which represent configurations of buckling forms of various possible types in connection with the corresponding changes in critical forces. Graphs of the dependence of the second critical force of the studied rods on their length are also given. It has been demonstrated that under certain conditions, reducing the length of the rod can lead to a reduction in its critical force.

Keywords:. compressed rod, change of critical force, buckling form, effect of length change, qualitative sign.

ЧИСЕЛЬНЕ ДОСЛІДЖЕННЯ ВПЛИВУ ДОВЖИНИ СТРИЖНЯ НА ЙОГО КРИТИЧНІ СИЛИ

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Анотація: Чисельно досліджується вплив зміни довжини поздовжньо стиснутого стрижня на його критичні сили. Дослідження виконується на прикладі двопрогінного прямолінійного стрижня постійної за довжиною згінної жорсткості, який стискається постійною по довжині поздовжньою силою і шарнірно спирається на одному з кінців на абсолютно жорстку опору, а всередині – на опору скінченої жорсткості. Зміна довжини стрижня відбувається за рахунок переміщення кінцевої шарнірної опори з відповідним збільшенням або зменшенням прилеглої ділянки стрижня без зміни положення і характеристик решти закріплень. Досліджується залежність критичних сил стержня від положення цієї опори і, відповідно, від довжини прилеглої стиснутої ділянки стержня. Розрахунки виконуються на основі використання відомих точних аналітичних виразів функцій впливу стрижня постійного поперечного перерізу, стиснутого постійною по довжині поздовжньою силою. У розглянутих окремих прикладах виявлено і цілком підтверджено якісні ознаки зростання, зменшення та екстремума простих критичних сил при зміні довжини стержня, пов'язані з якісними особливостями відповідних форм втрати стійкості, встановлені раніше теоретично. Зокрема, точними розрахунками продемонстровано, що зростання або зменшення простої критичної сили при зміні довжини ділянки стержня, що прилягає до переміщуваної опори, визначаються видом відповідної форми втрати стійкості в околі цієї опори. Розглядаються різні можливі конфігурації форм втрати стійкості та поведінка критичних сил при зміні довжини стержня для кожної з конфігурацій. З метою верифікації раніше встановлених теоретичних результатів, які стосуються вивчення поведінки не тільки основних критичних сил, а й вищих простих критичних сил, які мають довільний номер в спектрі, в статті розрахунки проводяться для других критичних сил стрижней, розглянутих у наведених прикладах. Результати проведених розрахунків продемонстровані у вигляді графіків, які представляють конфігурації форм втрати стійкості різних можливих типів у зв’язку з відповідними змінами критичних сил. Також наведені графіки залежності другої критичної сили досліджуваних стержней від їх довжини. Продемонстровано, що за певних умов зменшення довжини стержня може привести до зменшення його критичної сили.

Ключові слова: стиснутий стержень, зміна критичної сили, форма втрати стійкості, вплив довжини, якісна ознака.

1 INTRODUCTION

When designing and operating engineering structures containing longitudinally compressed elements, ensuring their stability is of great importance. In this case, as a rule, it is accepted, often without proper verification, that a shorter rod is more stable, i.e. has a higher critical force at which its loss of stability occurs. Although in most cases this assumption is satisfied, the issue of the relationship between the length of a compressed rod and its stability deserves more careful study, since engineering decisions made on its basis can lead to accidents or serious disruptions to the operation of the structure, making its safe operation impossible.

2 LITERATURE ANALYSIS AND PROBLEM STATEMENT

Under standard support conditions, the critical forces of longitudinally compressed rods, as a rule, decrease with increasing their length, i.e. when adding additional compressed sections at the ends of the rod and transferring the end supports to the ends of the formed elongated rod. However, as noted in a number of studies [1 – 3], with some ways of support, in particular in the presence of elastic pinches and/or intermediate elastic supports, reducing the length of the rod can lead to a decrease in critical forces and the risk of loss of stability. In this regard, of great theoretical and practical interest is the question of determining the conditions for the increase or decrease of the critical forces with a change in the length of the rod, as well as of determining the optimal length of the rod at which its critical forces reach its extremal value. This issue was subjected to a fairly detailed theoretical study in the author's work [3], where a straight rod was considered, hinged at one of the ends on an absolutely rigid support, and the change in length occurred due to the moving of this support and the corresponding lengthening or shortening of the adjacent section of the rod. The disadvantage of the work [3] is that, despite a fairly detailed theoretical justification of the results obtained in it, they were not illustrated with specific examples. At the same time, since these results are neither trivial nor obvious, it would be very desirable to provide examples that would confirm their validity.

3 THE PURPOSE AND OBJECTIVES OF THE STUDY

The purpose of the work is to numerically study the behavior of the critical force of a compressed rod when its length changes and to determine the signs of growth, decrease and extremum of the critical force using the example of a two-span rod of constant cross-section along the length, compressed by a constant longitudinal force along the length and supported by an intermediate support of finite rigidity. Note that the results of work [3] were established for a rod with variable bending stiffness along the length without limitations on the number and rigidity of intermediate supports. In addition, since in [3] a change in a critical force arbitrary in number in the spectrum was considered, the present article examines the behavior of the second critical force as a task that is less trivial compared to the case of the main critical force.

4 RESEARCH RESULTS

4.1. Preliminary results. First, we present the main results of the article [3], in which their proof can be found.

We consider a rod hinged at one of its ends on a rigid support and compressed by a longitudinal force constant along its length (at least within a certain area adjacent to this support). Forms of buckling that correspond to its critical forces (of any number) can be of five types (see Fig. 1).

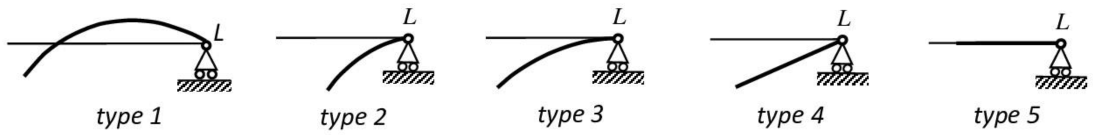

Fig. 1. Different types of buckling forms

The form of the 1st type in the neighborhood of the end support is concave towards the axis of the undeformed rod. In the type 2 form, the convexity faces this axis. Type 3 form has a horizontal tangent at the end support. Forms of the 4th and 5th types in some area adjacent to the end support have straight segments, i.e. are semi-curved. Examples of the implementation of forms of all five types are considered in [1 – 9]. The following six theorems establish qualitative signs of the increase or decrease of critical forces with a change in the length of the rod.

Theorem 1. A simple critical force, which corresponds to the form of the 1st type, decreases as the rod lengthens, provided that the reaction of the support is opposite to the deflections of the rod in the neighborhood of the support.

Theorem 2. The simple critical force corresponding to the type 2 form increases as the rod lengthens.

Theorem 3. Simple critical force, which corresponds to the form of the 3rd type,

a) decreases both when the rod is shortened and when the rod is lengthened, if, with rigid clamping of the end section, its number in the spectrum does not change and it remains simple,

b) increases both with shortening and lengthening of the rod, if this number decreases and it remains simple,

c) increases with shortening and decreases with lengthening of the rod, if it becomes double (i.e., after pinching, it corresponds to two numbers - the same one and one less).

Theorem 4. The simple main critical force when moving the end support reaches a maximum if it corresponds to a type 3 shape.

Theorem 5. The simple critical force, which corresponds to the form of the 4th type (half-curved), increases with elongation and decreases with shortening of the rod.

Theorem 6. The simple critical force, which corresponds to the type 5 form (half-curved), does not change when the rod is lengthened or shortened.

4.2. Numerical verification. We will demonstrate the validity of the established results using the example of a rod with bending rigidity EJ constant along its length, reinforced with an intermediate elastic support (Fig. 3 *a*).

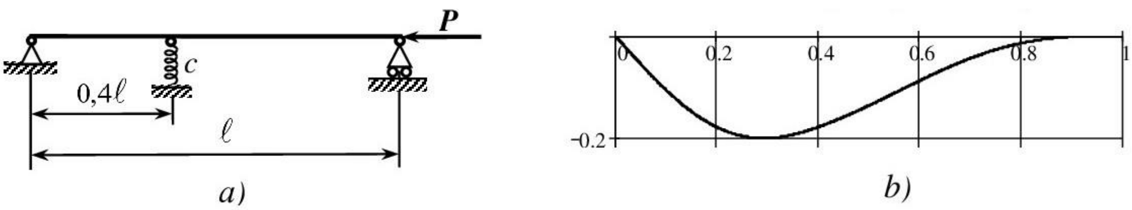

Fig. 2. A rod, the second form of which is the form of the 3rd type when $c = 5{,}605 \cdot c_0$, $P_2 = 5{,}120 \cdot P_E$

For this case, explicit expressions for the influence functions of the compressed rod and exact equations of critical forces for standard support conditions are known [10], using which we find that when the support is installed at a distance of $0,4\ell$ from the left support and with its rigidity equal to $c = 5,605 \cdot c_0$, where $c_0 = \pi^3 EJ/\ell^3$, the second critical force is equal to $P_2 = 5,120 \cdot P_E$, where $P_E = \pi^2 EJ/\ell^2$, and it corresponds to the form of the 3rd type (Fig. 3 *b*). As calculations show, the 2nd critical force of a single-span rod, hinged at one end and rigidly clamped at the opposite end, is equal to $6,047 \cdot P_E$. When introducing an intermediate support, it will become larger. It follows that the rigid clamping of the right end of the rod in Fig. 2 *a* will make his second critical force first, i.e. we are dealing with case *b*) of Theorem 3.

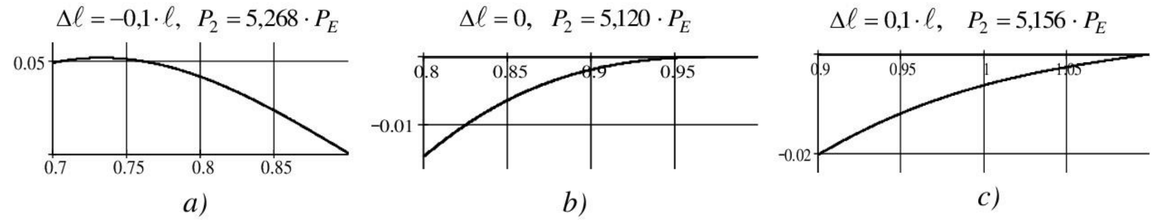

Fig. 3. The end fragment of the 2nd form of the rod shown in Fig. 2 a, when changing its length

Fig. 3 shows the end fragments of the second forms of the original rod, as well as rods shortened and extended by $\Delta\ell = 0,1\ell$ (of types 1 and 2, respectively). Nearby are indicated the corresponding values of the 2nd critical force, exceeding $P_2 = 5,120 \cdot P_E$, in full accordance with Theorem 3.

The elongation of the original rod is accompanied by an increase in the 2nd critical force until it reaches a maximum equal to $P_{2\max} = 5,185 \cdot P_E$, at a length equal to $1,184\ell$. The corresponding forms are shown in Fig. 4.

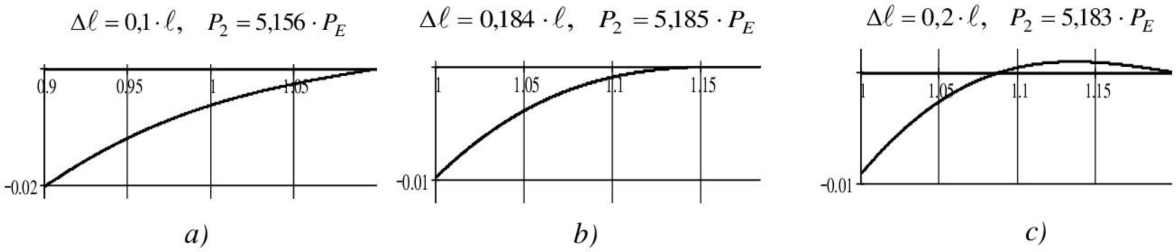

Fig. 4. Perturbations of the second buckling form of the 3rd type of rod shown in Fig. 3 a, with a length equal to $1,184\ell$

Note that now a single-span rod clamped at one end has a second critical force equal to $6,047 \cdot P_E / 1,184^2 = 4,317 \cdot P_E$, so that the considered critical force achieved as a result of installing an intermediate support will be the second in the spectrum and, thus, case *a*) of Theorem 3 is realized.

Graph of change in P_2 with change in length for the rod in Fig. 3 *a* is shown in Fig. 5 *a*, where the elongation of the rod is plotted horizontally in fractions of the original length ℓ, and the ratio P_2/P_E is plotted vertically.

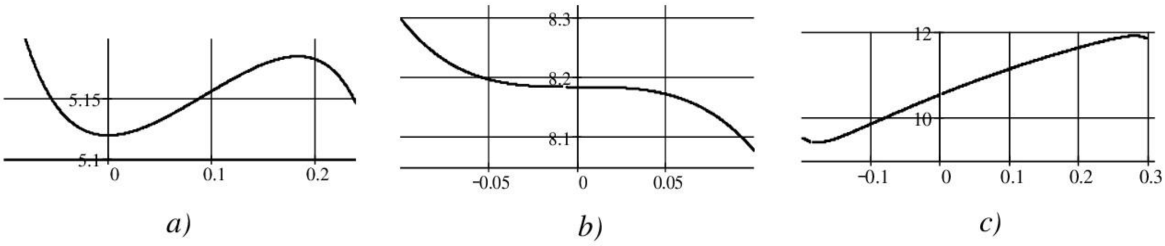

Fig. 5. Graphs of the dependence of the 2nd critical force on the length of the rod. The horizontal line shows the elongation in fractions of the length ℓ of the original rod, the vertical line shows the ratio P_2/P_E

To illustrate case *c*) of Theorem 3, consider a rod rigidly clamped at one end with an elastic support in the middle (Fig. 6 *a*).

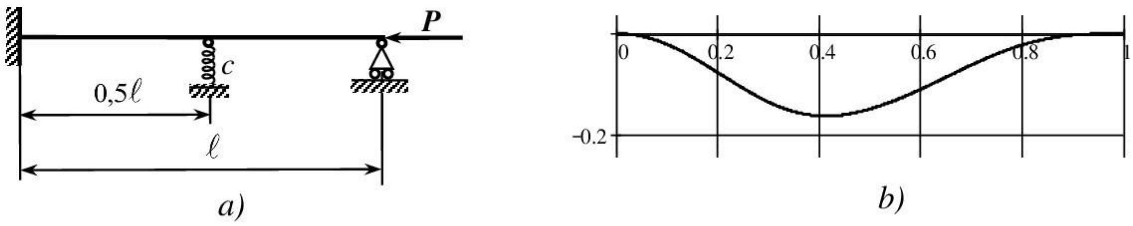

Fig. 6. Rod, the second buckling form of which is the form of the 3rd type when $c = 6,7 \cdot c_0$, $P_2 = 8,183 \cdot P_E$.

With a support stiffness coefficient equal to $c = 6,700 \cdot c_0$, the second buckling form has zero slope at the right end (see Fig. 6 *b*) and the clamping makes the corresponding critical force equal to $P_2 = 8,183 \cdot P_E$, double (the intermediate support is in the node of the second buckling form of a single-span rod rigidly clamped at both ends). The results of calculations of the 2nd critical force and the corresponding buckling forms when changing the length are presented in Fig. 7, from which it is clear that both elongation and shortening change the type of form from 3rd to 1st, which in this case, in accordance with Theorem 1, is a sign of a decrease in the critical force with increasing length of the rod.

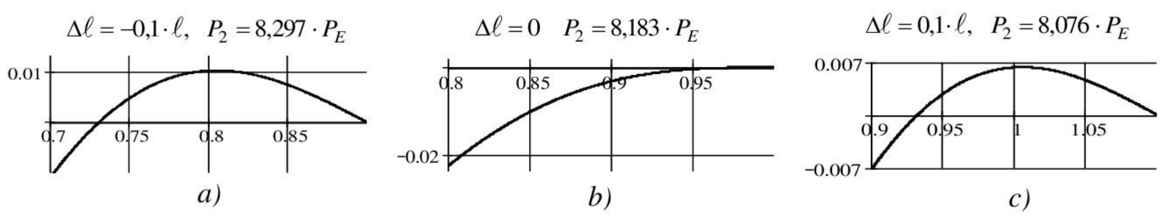

Fig. 7. The end fragment of the 2nd buckling form of the rod shown in Fig. 6 *a*, when its length changes

Fig. 5 *b* shows a graph of the dependence of the 2nd critical force for this case on the length of the rod. As can be seen, the behavior of the critical force completely follows Theorem 3.

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The 4th type buckling form is implemented for the rod shown in Fig. 6 *a*, with a support stiffness coefficient equal to $c = 11,233 \cdot c_0$, and a second critical force equal to $P_2 = 10,530 \cdot P_E$ (Fig. 8 b).

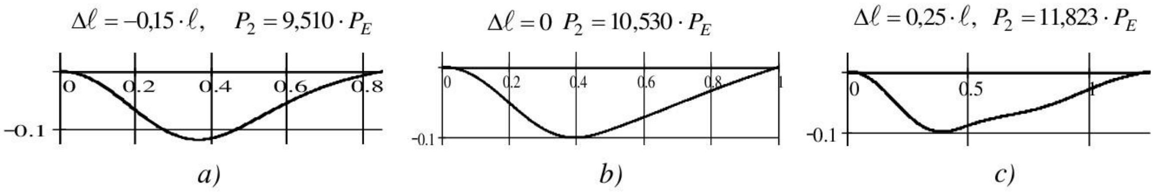

Fig. 8. Perturbations of the 2nd form of the 4th type of rod shown in Fig. 7 a, when its length changes, $c = 11,233 \cdot c_0$

Fig. 8 *a* and *c* also show the buckling forms of rods shortened by $\Delta\ell = 0,15 \cdot \ell$ and extended by $\Delta\ell = 0,25 \cdot \ell$, both of the 2nd type, which, according to Theorem 2, is a sign of an increase in the critical force along with the length of the rod, which is confirmed by direct calculation (the corresponding values P_2 are given above the curves). A graph of the dependence of P_2 on the length of the rod for this case is presented in Fig. 5 *c*.

All calculations and graphs are performed using Mathcad based on exact equations of critical forces, obtained on the basis of exact analytical expressions of the influence functions of a compressed prismatic rod supported by an intermediate elastic support. Forms in Fig. 2 – 8 are normalized so that the reaction of the elastic support is equal to 1. The ordinates of the forms are showed in fractions of the magnitude $1/c_0 = \ell^3 / \pi^3 EJ$.

5 RESEARCH RESULTS DISCUSSION

The results of calculations performed for specific examples presented in the article fully confirm the theoretical conclusions established earlier in the article [3]. Their validity has been established for various combinations of parameters of the considered models, in which different variants of the behavior of critical forces are realized when the length of the rod changes

6 CONCLUSIONS

The work obtained results that allow a deeper and more complete understanding of the behavior of the critical forces of a straight rod, hinged at one of the ends on a rigid support, when the length of the rod changes due to change of the fragment adjacent to this support. It has been established that the behavior (increase or decrease) of simple critical forces when the length of the rod changes is associated with the configuration of the corresponding forms of buckling in the neighborhood of the moving support. The results obtained can be used in the design and operation of engineering structures containing elements subject to longitudinal compression.

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8 ETHICAL DECLARATIONS

The proposed publication does not pursue any material interests and is intended only to expand and deepen knowledge concerning some topical problems of mechanics, and their sharing among specialists in relevant scientific fields.

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