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STABILITY OF RODS WITH INITIAL IMPERFECTIONS IN THE FORM OF ECCENTRICITY OF LOAD APPLICATION UNDER LINEAR AND NON-LINEAR CREEP CONDITIONS

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Abstract. Stability of a compressed rod having initial imperfections in the form of eccentricity of applied load under conditions of linear and nonlinear creep is considered. It is noted that all real elements have some initial imperfections in the form of technological deflections, eccentricities of applied loads, etc., so they begin to bulge from the very beginning of loading.

Another important factor in stability theory is the consideration of material creep. In this regard, the loading process is divided into two phases: the instantaneous loading process and the creep phase under constant external load. Moreover, creep can be time-limited or unrestricted.

In the paper formulas for determination of critical forces of stability loss of the rod having initial imperfections, under short-term and long-term action of load are obtained. The equation allowing to determine time of the first crack appearance is derived. Derived are equations the roots of which are loads at action of which the first cracks appear at initial moment of time and at arbitrarily long period of load action. Analysis of acting force determining the character of rod deformation is executed. From the constructed stability equation it is possible to determine the critical force corresponding to the critical length of the section with cracks.

For similar problems in nonlinear formulation formulas for determining critical force and critical displacement corresponding to maximum load are obtained. For the case of long duration load the equation which establishes relationship between load and displacement is obtained. Equation for determination of critical force under prolonged action of load has been derived. It has been established that critical displacement is the same under short- and continuous action of load. It is shown that at any intermediate moment critical displacement can be achieved under load lying in certain interval.

Keywords: stability, rod, initial imperfection, eccentricity, linear creep, non-linear creep, critical force, crack, critical displacement.

СТІЙКІСТЬ СТРИЖНІВ, ЩО МАЮТЬ ПОЧАТКОВІ НЕДОСКОНАЛОСТІ У ВИГЛЯДІ ЕКСЦЕНТРИСИТЕТУ ДОДАТКУ НАВАНТАЖЕННЯ В УМОВАХ ЛІНІЙНОЇ ТА НЕЛІНІЙНОЇ ПОЛУЧІСТЬ

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Анотація. Розглядається стійкість стисненого стрижня, що має початкові недосконалості у вигляді ексцентриситету застосування навантаження в умовах лінійної та нелінійної повзучості. Зазначається, що це реальні елементи мають ті чи інші початкові недосконалості як технологічних прогинів, ексцентриситетів прикладених навантажень та інших, тому вони починають витріщатися від початку навантаження.

Ще одним важливим фактором теорії стійкості є облік повзучості матеріалів. У зв'язку з цим процес навантаження розділяється на два етапи: миттєвий процес навантаження та етап повзучості при постійному зовнішньому навантаженні. Причому повзучість може бути обмеженою в часі або необмеженою.

У роботі отримані формули визначення критичних сил втрати стійкості стрижня, має початкові недосконалості, при короткочасному і тривалому дії навантаження. Побудовано рівняння, що дозволяє визначити час появи першої тріщини. Виведені рівняння, корінням яких є навантаження, при дії яких утворюються перші тріщини в початковий момент часу і при будь-якому великому часі дії навантаження. Проведено аналіз чинної сили, що визначає характер деформування стрижня. Зі побудованого рівняння стійкості можна визначити критичну силу, якій відповідає критична довжина ділянки з тріщинами.

Для аналогічних завдань у нелінійній постановці отримані формули визначення критичної сили і критичного переміщення, відповідальних максимальному навантаженні. Для випадку тривалої дії навантаження одержано рівняння, що встановлює зв'язок між навантаженням та переміщенням. Виведено рівняння визначення критичної сили при тривалому дії навантаження. Встановлено, що критичні переміщення однакові за короткочасної та тривалої дії навантаження. Показано, що у будь-який проміжний момент часу критичне переміщення може бути досягнуто при навантаженні, що лежить у певному інтервалі.

Ключові слова: стійкість, стрижень, початкова недосконалість, ексцентриситет, повзучість лінійна, повзучість нелінійна, критична сила, тріщина, критичне переміщення.

1 INTRODUCTION

It is known [1, 2] that one of the most important problems of deformable solid mechanics is the problem of structural stability. It is the loss of stability that is associated with a number of accidents and disasters. Loss of stability is especially dangerous because it occurs suddenly, often with stresses that are significantly lower than the ultimate strength of the material.

Modern stability theory is based on the study of the loading process of structures and their elements, and this process is considered unstable if a catastrophic development of movements and deformations corresponds to its continuation, however small. Failure occurs at the limit points called bifurcation points and the corresponding loads are called stability limits or critical loads.

All real elements have some kind of initial imperfection (technological deflections, eccentricity of load application etc.) and therefore they start to bulge from the very beginning of loading.

Another important factor in stability theory is the consideration of material creep. The loading process is therefore divided into two phases: the instantaneous loading process and the creep phase under constant external load. Moreover, creep can be time-limited or unrestricted.

2 LITERATURE ANALYSIS AND PROBLEM STATEMENT

The problem on the stability of an elastic rod under the action of an axial compressive force was first solved by L. Euler. This solution is given in numerous literature on the stability of rods, of which special mention should be made [3-5]. However, field tests have shown that this solution is not applicable for real steel rods, due to the inevitable curvature of the element axis during fabrication and transportation and inaccuracies in alignment during assembly. In this regard, different solutions have been obtained for a rod under the action of an axial compressive force applied with eccentricity [3-5].

An interesting analysis of the calculation procedures for centrally compressed steel rods laid down in various normative documents has been carried out in [6].

One of the first publications in which the influence of initial imperfections on stability was investigated is the monograph by A. R. Rzhanitsyn [7]. Initial geometric imperfections most significantly affect the stability of thin-walled open section elements [8]. In [9] a fourth degree polynomial is used to describe the shape of initial imperfections.

There are a large number of publications on creep rupture of compressed rods, including [10-14] and others. The approaches to the problem are very different - finite difference method, finite element method, Bubnov-Galerkin method, power method in the Ritz-Timoshenko form, etc.

N. Rabotnov [15] and S. A. Shesterikov [16, 17] suggested a new approach - they connected the question of creep stability with the classical definition of stability. Taking strengthening law as a basis, they conducted its linearization taking into account small deflections, and then performed analysis of rod motion under action of perturbations.

The works of scientists from Odessa school of creep theory headed by I. E. Prokopovich [18-20] should not be overlooked.

All works considering the issues of rod stability with regard to creep can be divided into two directions. The first, classical approach assumes the existence of change of stable configurations of equilibrium - after some time, which is called critical, there is a transition of rectilinear form into curved one. The second approach to investigating buckling is to assume that the creeping process in the rod leads to reduction of its stiffness and, consequently, the loss of stability.

The second approach in investigation of rod bulging in creep is based on the consideration of initial imperfections, defects (malformations, eccentricity of load, etc.). In this approach it is assumed that initial imperfections in geometry or eccentricities increase with time, leading to failure.

3 PURPOSE AND OBJECTIVES OF THE STUDY

The aim of this work is to solve the problem on the stability of a rod having initial imperfections in the form of eccentricity of load application under conditions of linear and nonlinear creep.

In the stability of elastic rods one distinguishes a loss of stability of the first kind, associated with the possibility of existence of two forms of equilibrium - stable and unstable, and a loss of stability of the second kind, associated with the possibility of unlimited development of movements of the rod, possessing one or another initial imperfection.

Since the creep increases the deformations and displacements, it is natural to consider a loss of stability of the second kind in the study of the stability of rods made of materials with a considerable creep. For a rectilinear rod, such a loss of stability is possible only in the presence of initial imperfections of shape (initial failure) or state (eccentric application of compressive force, deviation from rectilinear shape due to external action).

4 RESEARCH RESULTS

To solve the problem of the stability of a flexible reinforced concrete rod under conditions of linear creep with account of cracking, consider a rod pivoted at the ends with a rectangular symmetrically reinforced cross-section. The load P is constant in time and is applied with eccentricity e_0.

Two stages can be distinguished in the deformation of such a rod. Stage I - the load P is conditionally "small" to such an extent that cracks in the concrete tensile zone do not appear during the whole considered time interval. Stage II - the load P is conditionally "large" to such an extent that cracks in the concrete tensile zone appear either at the time t_0 of load application or at the time $t_I > t_0$ (t_I - time of the first crack formation).

The relationship between deformations and stresses in concrete is established by the linear theory of elastic heredity (TEH):

$$\varepsilon(t) = \sigma(t)\delta(t,\tau) - \int_{t_0}^{t} \sigma(\tau)\delta(t,\tau)/\delta\tau d\tau;$$
$$\delta(t,\tau) = \frac{1}{E(\tau) + C(t,\tau)}; \tag{1}$$
$$C(t,\tau) = C_0\left[1 - Be^{-\gamma(t-\tau)}\right];$$
$$C(t_0,t_0) = C_0(1-B).$$

The difference $(1-B)$ takes into account the fast-moving part of the creep deformation, conventionally referred to a point in time t_0, hence $C(t_0,t_0)$ corresponding to the short-term action of the load.

Stage I. At this stage the solution of the integrodifferential equation of motion of the reinforced rod, or a corresponding partial differential equation, is the function

$$y(z,t)=\frac{f(t)\sin\pi x}{e_0}, \tag{2}$$

where $f(t)$ — displacement of the middle section along the length of the bar:

$$\begin{aligned} &f(t)=f(t_0)\{[1-(P_{cr}-P)/(P_{cont}-P)]\cdot \\ &\cdot\exp[-\gamma_1(P_{cont}-P)/(P_{cr}-P)(t-t_0)]+(P_{cr}-P)(P_{cont}-P)\}; \end{aligned} \tag{3}$$

$$f(t_0)=\frac{4e_0P}{\pi(P_{cr}-P)};$$

$$\gamma_1=\frac{\gamma(1+\varphi)}{[1+(1-B)\varphi]};$$

$$\varphi=E_bC_0.$$

At $t-t_0\to\infty$ we have:

$$f(\infty)=\frac{4e_0P}{\pi(P_{cont}-P)}. \tag{4}$$

According to the solutions obtained for the reinforced concrete core, two forces can be specified – P_{cr} and P_{cont}.

$$\begin{aligned} &P_{cr}=\pi^2E_0J_0/l^2\{\alpha+1/[1+(1-B)\varphi]\}; \\ &P_{cont}=\pi^2E_bJ_b/l^2[\alpha+1/(1+\varphi)]; \\ &\alpha=\mu\rho_1\eta; \\ &\mu=\frac{2A_s}{A_b}; \\ &\rho_1=\frac{h_1^2A_b}{I_b}; \\ &\eta=\frac{E_s}{E_b}. \end{aligned} \tag{5}$$

The critical force for a loss of stability of the second kind is defined as the minimum value of force that results in an unrestricted increase in displacement.

P_{cr} – the critical force under short-term loading, determined by the condition $f(t_0)\to\infty$;

P_{cont} – the critical force under continuous load, determined by the condition $f(\infty)\to\infty$.

Knowing the displacement $y(z,t)$, it is possible to determine the height of the concrete compression zone and the stresses in the reinforcement and concrete in any cross-section at any time.

In bendable and eccentrically compressed reinforced concrete bars, cracks in the cross-sections in the tensile concrete appear if the condition

$$\sigma_{b(z,t)}=-2R_p. \tag{6}$$

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After a number of transformations we obtain an equation which allows us to determine the time of the appearance of the first crack. The first crack appears in the average section along the length of the rod $(z = l/2)$, and the time of its appearance is defined as follows:

$$t_T = t_0 - 1/\gamma_1 (P_{cr} - P)/(P_{cont} - P)\cdot \\ \cdot \ln\left[(P_{cr} - P)\cdot(P_{cont} - P)/(P_{cont} - P_{cr})/\pi y_T / 4e_0 P - 1/(P_{cont} - P)\right]. \quad (7)$$

Of interest are the loads P_T and P_T^*, under the action of which the first cracks are formed at the times $t = t_0$ and $t = \infty$, respectively.

P_T and P_T^* are defined as the roots of the equations

$$\frac{\pi y_T (P_{cr} - P)}{4e_0 P} = 1; \\ \frac{\pi y_T (P_{cont} - P)}{4e_0 P} = 1. \quad (8)$$

Let's look at the magnitude of the acting force, which determines the nature of the deformation of the rod.

If $P \le P_T^*$, then over the whole considered time interval the deformation occurs without crack formation (stage I). If $P_T^* < P < P_T$, then cracks appear during the deformation process (stage II). If $P > P_T$, then cracks appear immediately after load application (Stage II).

Stage II. In the section of the rod with length l_T there will be additional displacements caused by the decrease of stiffness as a result of cracking. Considering the rod deformation at the segment l_T, one can find the critical force under prolonged action taking into account cracking - P_{cont}^T.

After a number of transformations the stability equation is reduced to the form

$$P\cdot l_T = 2\pi^2 A_s E_s \left[3(y_T + e_0)^2 - h(y_T + e_0) + h_1^2\right]. \quad (9)$$

Here l_T – the length of the section with cracks, determined by the dependencies

$$\frac{\pi y_T (P_{cr} - P)}{4e_0 P} = 1; \\ \frac{\pi y_T (P_{cont} - P)}{4e_0 P} = 1; \\ l_T = l\left\{1 - 2/\pi \arcsin\left[\pi y_T (P_{cont} - P_{cont}^T)/4e_0 P_{cont}^T\right]\right\}. \quad (10)$$

To the critical force P_{cont}^T, determined from equation (9), corresponds the critical length of the section with cracks $l_{T_{cont}}$.

5 DISCUSSION OF RESEARCH FINDINGS

It can be concluded from the results of these calculations that consideration of creep and cracking leads to a significant reduction in the critical forces.

However, all the above considerations refer to the behavior of compressed flexible rods under prolonged action of loading under conditions of linear creep. Let's consider the operation of the rod under creep conditions in geometrically nonlinear formulation.

Consider a flexible rod made of a material with creep. The support is articulated. The rod is loaded with a longitudinal compressive force P, constant in time, applied with eccentricity e_0 in the direction of displacement.

The integral-differential equation of slow motion has the form

$$\frac{1}{\rho(x,t)}-\frac{P}{EI}\left[y(x,t)+e_0\right]+\frac{P}{I}\int_{t_0}^{t}\left[y(x,\tau)+e_0\right]\frac{\partial\sigma(x,\tau)}{\partial\tau}d\tau=0. \tag{11}$$

Here $y(x,\tau)$ – The movement of the rod in the plane of deformation (in the direction of e_0); $1/\rho(x,t)$ – rod curvature in the same plane;

$$\frac{1}{\rho(x,t)}=-y^4(x,t)\left[1+(y')^2(x,t)\right]^{-\frac{1}{2}}. \tag{12}$$

The creep is further accounted for at the level of elastic heredity theory (TEH), as in the case of the linear creep variant.

After a number of transformations the equation is obtained

$$f^3(t_0)+\frac{8l^4}{3\pi^4}\left(\frac{P}{EI}-\frac{\pi^2}{l^2}\right)f(t_0)+\frac{32e_0}{3\pi}\cdot\frac{l^4}{\pi^4}\cdot\frac{P}{EI}=0, \tag{13}$$

which establishes the relationship between displacement, eccentricity and short-term loading.

After introducing relative eccentricities $s=e_0/l$ and displacements

$$F(t_0)=\frac{\pi}{e_0}f(t_0);\quad F(t)=\frac{\pi}{e_0}f(t), \tag{14}$$

equation (13) is written as:

$$F^3(t_0)+\frac{8}{3}\left(\frac{P}{EI}-1\right)F(t_0)+\frac{32}{3}\cdot s\cdot\frac{P}{P_e}=0. \tag{15}$$

Here $P_e=\pi^2EI/l^2$ – is the Euler force, i.e. the critical force under short-term loading in the case of approximate curvature (linear formulation).

From equation (15) it follows a linear relationship

$$P=\frac{1}{8}P_e\frac{8F(t_0)-3F^3(t_0)}{4s+F(t_0)}, \tag{16}$$

which establishes the relationship between load, displacement and eccentricity. The critical force is determined from the condition

$$\frac{\partial P}{\partial F}=0. \tag{17}$$

It follows that

$$F^3(t_0)+6sF^2(t_0)-\frac{16}{3}s=0. \tag{18}$$

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The critical force is determined by the formula:

$$P_{cr} = \frac{1}{8} P_e \frac{F_{cr}\left(8 - 3F_{cr}{}^2\right)}{4s + F_{cr}}, \tag{19}$$

where F_{cr} –the displacement corresponding to the maximum load, which is the root of equation (18):

$$F_{cr} = 2\sqrt[3]{\frac{1}{3}s\left(1 - 3s^2 + \sqrt{1 - 6s}\right)} + 2\sqrt[3]{\frac{1}{3}s\left(1 - 3s^2 - \sqrt{1 - 6s}\right)} - 2s. \tag{20}$$

At small values of s $(s \le 0{,}01)$ dependency can be used:

$$F_{cr} = 2\left[\sqrt[3]{\frac{1}{3}s\left(1 - 3s^2 + \sqrt{1 - 6s^2}\right)} - s\right]. \tag{21}$$

Displacements at $t \to \infty$ are defined as a root of equation

$$F^3(\infty) - \frac{8}{3}\left[1 - \frac{P}{(1+\varphi)P_e}\right]F(\infty) + \frac{32}{3}s\frac{P}{(1+\varphi)P_e} = 0. \tag{22}$$

From (22) it follows

$$P = \frac{1}{8}(1+\varphi)P_e \frac{8F(\infty) - 3F^3(\infty)}{4s - F(\infty)}. \tag{23}$$

This relationship establishes the relationship between load and displacement in the case of prolonged load action. This is the equilibrium curve. The maximum on this curve determines the critical force P_{cont}. From condition (17) it follows that

$$F^3(\infty) + 6sF^2(\infty) + \frac{16}{3}s = 0. \tag{24}$$

Critical force P_{cont} is defined by equation:

$$P_{cont} = \frac{1}{8}(1+\varphi)P_e \frac{F_{cont}\left(8 - 3F_{cont}{}^2\right)}{4s - F_{cont}}, \tag{25}$$

where F_{cont} – the displacement corresponding to the maximum load, which is the root of equation (24).

It is obvious that the roots of equations (18) and (24), representing the critical displacements F_{cr} and F_{cont} of both short-term and long-term loading respectively, are the same

$$F_{cr} = F_{cont} = F^*. \tag{26}$$

In case $t = t_0$ displacement F^* is developed by the action of a force F_{cr}, and in case $t \to \infty$ – by the action of a force F_{cont}.

Since the critical displacements for both momentary and continuous loading are the same, it follows that at any intermediate point in time $t_0 \le t \le \infty$ such a displacement F^* can be achieved at a load lying in the interval

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$$F_{cr} > F > F_{cont} . \tag{27}$$

6 CONCLUSIONS

Thus, formulas for determination of critical forces of stability loss of the rod having initial imperfections, under short-term and long-term action of load P_{cr} and P_{cont} respectively, are obtained. An equation permitting to determine time of the first crack to appear has been derived. Derived are the equations the roots of which are loading P_T and P_T^*, under the action of which the first cracks form at time moments $t = t_0$ and $t = \infty$ respectively. The analysis of acting force determining the character of rod deformation has been carried out. From the stability equation (9) it is possible to determine the critical force P_{cont}, to which corresponds the critical length of the section with cracks $l_{T_{cont}}$.

For the similar problems in nonlinear formulation formulas for determining of critical force and critical displacement corresponding to maximum load have been obtained. For the case of prolonged action of load, equation which establishes relationship between load and displacement was obtained. Equation for determination of critical force P_{cont} under prolonged action of load has been derived. It has been established that critical displacement is the same in short term and long term action of load. Consequently, at any intermediate point of time $t_0 \le t \le \infty$ displacement F^* can be achieved at a load lying in the interval of $F_{cr} > F > F_{cont}$.

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