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ON THE OPTIMAL POSITION OF THE INTERMEDIATE SUPPORT OF THE COMPRESSED THREE-SPAN ROD AND ITS QUALITATIVE FEATURES

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Abstract: The optimization problem is considered, which consists in maximizing the main critical force of a three-span longitudinally compressed rod supported at the ends on absolutely rigid hinge supports due to the optimal choice of the position of one of the intermediate supports. It is known that in many cases this position is a node of the buckling form, which corresponds to the second critical force in the spectrum of the two-span rod formed by removing the moving support. A range of recent studies have described cases where the maximum critical force is reached at other positions. This, in particular, occurs at a finite stiffness of one or both end supports of the rod. The proposed work seeks the optimal position of the rigid intermediate support, provided that the second intermediate support has a finite stiffness and a fixed position. The compressive force is assumed to be constant along the length of the rod, bending stiffness can vary according to the length of the rod according to arbitrary way. It is established that under certain conditions the solution of this problem can be reduced to the solution of another, previously studied problem, which seeks the maximum critical force of a two-span rod by changing its length, at which some segment of the rod adds or removes at one end of the rod with the transfer of the corresponding hinged support at the end of the newly created rod. The paper founds and describes the characteristic qualitative features of the buckling forms, which correspond to the maximum of the main critical force, in particular the absence of deformation of the bending of the end span adjacent to the moving support. The limitations in which the approach proposed in the paper leads to the determination of the desired optimal position of the movable support are studied. The results are obtained mainly on the basis of the systematic use of qualitative methods and allow to obtain qualitative estimates for the localization of the moving support and the value of the corresponding critical force. An example illustrating the proposed approach and the reliability of the results of its application are considered.

Keywords: compressed rod, critical force, optimization, buckling form, length effect, qualitative sign.

ПРО ОПТИМАЛЬНЕ ПОЛОЖЕННЯ ПРОМІЖНОЇ ОПОРИ СТИСНЕННОГО ТРИПРОЛІТНОГО СТЕРЖНЯ ТА ЙОГО ЯКІСНІ ОСОБЛИВОСТІ

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Анотація: У роботі розглядається задача оптимізації, яка полягає у максимальному підвищенні основної критичної сили трипролітного шарнірно опертого по кінцях на абсолютно жорсткі шарнірні опори поздовжньо стисненого стрижня за рахунок оптимального вибору положення однієї з проміжних опор. Відомо, що у багатьох випадках таким положенням є вузол форми втрати стійкості, що відповідає другій за номером критичній силі в спектрі двопрогонового стрижня, утвореного видаленням опори, що переміщується. У ряді досліджень останніх років були описані випадки, коли максимум критичної сили досягається при інших її положеннях. Це, зокрема, має місце при скінченій жорсткості однієї або обох кінцевих опор стрижня. У запропонованій роботі розшукується оптимальне положення жорсткої проміжної опори за умови, що друга проміжна опора має скінчену жорсткість та фіксоване положення.

Стискаюча сила передбачається постійною за довжиною стрижня, згинальна жорсткість може змінюватися за довжиною стрижня за довільним законом. Встановлено, що при певних умовах розв'язання цього завдання може бути приведено до розв'язання іншої, раніше дослідженої задачі, в якій розшукується максимум основної критичної сили двопрогонового стрижня за рахунок зміни його довжини, при якій на одному з кінців стрижня приєднується або видаляється деяка ділянка стрижня з перенесенням відповідної шарнірної опори у кінець новоствореного стрижня. В роботі виявлено і описано характерні якісні особливості форм втрати стійкості, які відповідають максимуму основної критичної сили, зокрема відсутність деформації вигину крайнього прольоту, що примикає до опори, яка переміщується. Вивчено обмеження, у яких запропонований у роботі підхід призводить до визначення шуканого оптимального положення переміщуваної опори. Результати отримано в основному на підставі систематичного використання якісних методів і дозволяють отримати якісні оцінки для локалізації опори, що переміщується, і значення відповідної критичної сили. Розглянуто приклад, що ілюструє пропонований підхід та достовірність результатів його застосування.

Ключові слова стиснутий стрижень, критична сила, оптимізація, форма втрати стійкості, вплив довжини, якісна ознака.

1 INTRODUCTION

Further abbreviations are used: CRF – critical force; BF – buckling form; 2nd BF – BF, which corresponds to the second CRF by number in the spectrum of CRFs.

For reliable operation of many engineering structures containing compressed elements, it is necessary to ensure their stability. In this regard, the task of finding ways to improve the stability of such elements, both at the design stage and during the operation of the structure, acquires great practical importance.

One of the ways to increase the stability of multi-span rods is to vary the position of their intermediate supports in order to increase their CRF. At the same time, the optimal positions of the movable supports, which provide the maximum of the main CRF of the rod, have some qualitative features. In particular, for a number of simple cases it has been established [1, 2] that the optimal position of the internal support is the node of the 2nd BF in the spectrum of the rod formed by the removal of the movable support. However, it cannot be guaranteed that this conclusion will be valid under various support conditions, just as it is impossible to guarantee the existence of a node at the 2nd BF of the rod. These circumstances prompted research, the results of which are presented in this paper.

2 LITERATURE ANALYSIS AND PROBLEM STATEMENT

As recent studies [3 – 7] have shown, under certain conditions, in particular, with finite stiffness of the extreme supports, the node of the 2nd BF, even if it exists, does not provide the maximum CRF. Under these conditions, the search for the optimal position requires a different approach and, in some cases, leads to the appearance of special semi-curved BFs with rectilinear horizontal sections. The cases considered so far have been limited to rods with absolutely rigid intermediate supports. At the same time, in practice, rods with elastic intermediate supports can be encountered, the optimization of which has its own features, and the study of these features is of great theoretical and practical interest. The results presented below refer to the solution of some of the problems that arise in this case. This solution is based on the use of qualitative methods.

3 THE PURPOSE AND OBJECTIVES OF THE STUDY

In this paper, the problem of determining the optimal position of an absolutely rigid intermediate support is considered using the example of a three-span rod (Fig. 1*a*), in which one intermediate support has a finite rigidity and a fixed position.

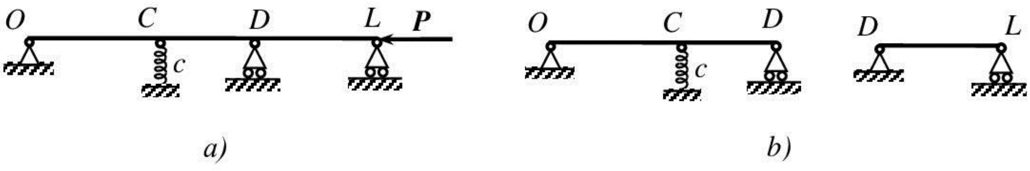

Fig. 1. The rod S to be optimized (*a*) and its components (*b*)

The ends of the rod are hinge supported on rigid supports. The distribution of bending stiffness along the length of the rod is assumed to be arbitrary. The compressive force in all sections is the same. We look for the optimal position of the rigid support. The optimal position is understood as such a position of the support, in which the main CRF of the rod reaches its maximum value.

4 RESEARCH RESULTS

4.1. Preliminary results. The following notations are used:

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(MN) is a single-span rod, hinged at the ends M and N on absolutely rigid supports; $P(MN)$ is the main CRF of the rod (MN), A is the node of the 2nd BF of the rod (OL), A_1 and A_2 are the nodes of the 3rd BF of the rod (OL), numbered from left to right;

Further, the following results are systematically used.

4.1.1. The imposition of one constraint on a rod system containing compressed elements cannot decrease any of its CRFs and cannot make the CRF higher than the next one by number in the spectrum of the system before the constraint is imposed [1].

4.1.2. Let us agree to say that a constraint is established in a generalized BF node or that a constraint is orthogonal to a BF, if the work of the reaction of the constraint on this BF is equal to zero.

The number of CRFs, which are strictly less than some CRF, does not change as a result of imposing a constraint, if this constraint is not orthogonal to at least one of the BFs corresponding to this CRF.

In order to the multiplicity of the CRF not to decrease after the introduction of the constraint, it is necessary and sufficient that this constraint be imposed in the generalized node of each of the BFs corresponding to this CRF, which, therefore, will be the CRF of the system formed after the constraint has been set. In particular, for a simple CRF to be preserved in the spectrum after a constraint is set, it is necessary and sufficient that the constraint is superimposed in the generalized node of the corresponding BF [1].

4.1.3. The reaction R of the intermediate hinged support is considered positive when it is directed upwards. The slope angle θ of the rod cross section is positive if it is turned clockwise. Then, for a sufficiently small displacement of the support to the right, the simple CRF increases if $R\theta > 0$, and decreases if $R\theta < 0$. If, at a certain position of the support, the CRF reaches an extremum, in this position $R\theta = 0$ [8].

4.1.4. The CRF of a two-span rod OD with an elastic intermediate support (Fig. 1 *b* on the left) changes with a change in the length of the segment with the transfer of the extreme hinged support to the end of the elongated or shortened rod. In [9], the existence of such a value c_{cr} of the stiffness coefficient of the intermediate support was established that at $c > c_{\text{cr}}$ the main CRF of the rod reaches a maximum at a certain length $|CD|$; this maximum is greater than $P(OC)$.

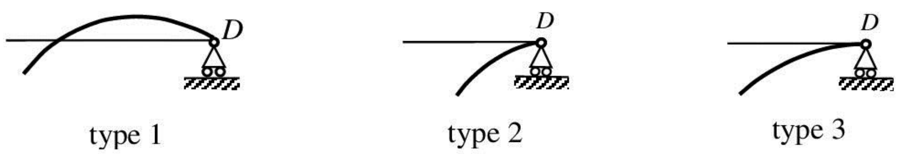

Fig. 2. Typical configurations of the BF of the rod at the end adjacent to the hinged support

The behavior of the main CRF of the rod OD with a change in its length is determined by the configuration of the end segment of the corresponding BF, adjacent to the movable support (Fig. 2), and this CRF with increasing length decreases for type 1 BF, increases for type 2 BF, and reaches a maximum for BF of the 3rd type, which has a zero slope of the end cross section. The value c_{cr} is equal to the stiffness coefficient of the elastic support, which provides the maximum increase in the main CRF of the two-span rod OD (Fig. 1*b* on the left), provided that C is the node of the 2nd BF of the rod (OD). It is defined by the equalities

$$P(OC) = P(CD) = c_{\text{cr}} \Bigg/ \left(\frac{1}{|OC|} + \frac{1}{|CD|} \right). \quad (1)$$

In particular, for a rod with the bending stiffness EJ constant along the length $c_{cr} = 2\pi^2 EJ / |OC|^3$. Generalizations of the results of [9] concerning the relationship between the CRFs and the length of the rod are presented in [10].

4.1.5. The main BF of a two-span rod, hinged with its ends on rigid supports, with an elastic intermediate support, has no nodes inside the segment containing a node of the 2nd BF of the rod, devoid of an intermediate support (Fig. 3) [11].

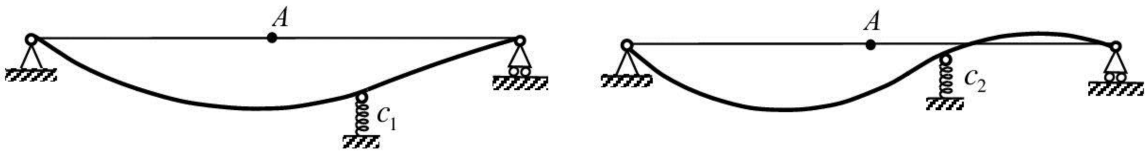

Fig. 3. Characteristic configurations of the main BF of a two-span rod. A – node of the 2nd BF of a single-span hinged rod, $c_1 < c_2$

4.2. Determination of the optimal position of the support D. Mentally position ourselves so that the elastic support C (see Fig. 1 *a*) is to the left of the node A of the 2nd BF of the rod (OL), and when searching for the optimal position of the support D, we restrict ourselves to the right segment CL, since it is less stable $\left(P(OC) > P(CL)\right)$.

4.2.1. **Approach idea.** The idea of finding the optimal position of the support D uses the mental division of the optimized rod into two parts (Fig. 1*b*). As noted in Sec. 4.1.4, at sufficiently high values of the stiffness coefficient c of the elastic support, the CRF of the left segment OD reaches its maximum value at a certain position D^* of the support D and, accordingly, at a certain length of the segment OD. If the cut D is located to the left of D^*, the CRF of the segment OD is less than its maximum. By moving the cut to the right, we will increase the CRFs of both the segments OD (until it reaches the maximum) and the right rod (DL), and it is natural to assume that the CRF of the rod S will also increase. When the moved cut is in the position D^* that ensures the maximum CRF of the segment OD, the BF of this segment will belong to the 3rd type (see Sec. 4.1.4, Fig. 2), and its smooth conjugation with the undeformed segment DL forms a semi-curved BF of the optimized rod S, since “splicing” segments OD^* and D^*L is a superimposition of a constraint orthogonal to the corresponding BF of a split rod (Sec. 4.1.2). As follows from Sec. 4.1.3, the corresponding position of the support D satisfies the necessary condition for the extremum of the CRF of the rod S. Further, some sufficient conditions are established under which this position ensures the maximum of the main CRF of the rod S. In what follows, we will call the point D^* a conjugation point or a singular point of a semicurved BF.

4.2.2. **Theorem**. If at $c > c_{cr}$ the main CRF corresponds to a semi-curved BF, then the optimal position of the support is the conjugation point of this BF.

The existence of a semi-curved BF for $c > c_{cr}$ follows from what was said in Sec. 4.2.1. Therefore, to prove the theorem, it suffices to prove that this BF corresponds to the maximum of CRF, if it is also the main one.

If the considered main CRF is a multiple, then (regardless of the configuration of BFs corresponding to it) the movement of the support can be carried out by first removing the support, thereby reducing the main CRF and not changing the 2nd CRF (Sec. 4.1.1), and then installing it in a new place, after whereby the main CRF becomes no higher than the initial multiple CRF, i.e. that this initial CRF is the maximum, which was to be proved.

Next, we consider the case when the considered CRF is simple.

First, let us prove that the installation of a movable support at the conjugation point of the BF provides a local maximum of CRF.

Let D^* be a singular point of a semicurved BF. Moving the support to the right of D^* can be done in two steps. On the first, keeping the support in position D^*, an additional support is placed on the right in a straight segment. This support does not distort the semicurved BF and does not change the corresponding main CRF of the rod S. At the second step, the support that was in D^* is removed, which leads to a decrease in the CRF. Thus, it has been established that when the support is displaced to the right of D^*, CRF decreases.

With a small displacement of the support to the left, we consider two cases: the first case is that the CRF of the rod (D^*L) is less than or equal to the CRF of the entire rod S, the second is that the CRF of the rod (D^*L) is greater than the CRF of the rod S.

In the first case, with a small displacement of the support D from the position D^*, a rod is formed, cutting which on the support D, we get two rods – OD and (DL), each of which has CRF less than the CRF of the original rod. By imposing a constraint that eliminates the cut, it is impossible to increase the main CRF above the second one in the spectrum (Sec. 4.1.1), whence it follows that as a result of the displacement, the CRF decreased.

In the second case, we note that the spectra and BFs of the rod S and the two-span rod OD (Fig. 1*b* on the left) continuously depend on the position of the support D. The same conclusion is also valid for the rod $\bar{S}$ formed from S by setting a rigid clamping, combined with the support D. Then there is such a neighborhood of the point D^* that when moving the support D to any of its points to the left of D^*

a) the main CRFs of rods S and $\bar{S}$ remain simple,

b) the main CRF of a two-span rod OD decreases when the support D moves to the left and it corresponds to a BF of the 2nd type,

c) CRF of rod (DL) remains greater than CRF of the rod S and the reaction of the displaced support at buckling of the rod S along the main BF remains directed downward, i.e. in the direction of the transverse displacement of the cross section C.

Let us consider a two-span rod OD (Fig. 1*b* on the left), introduce elastic clamping in its cross section D, and watch the change in its main CRF and the BF corresponding to it with a continuous increase in the clamping stiffness. The following statements are true.

I. At no finite value of the clamping stiffness, the slope of the cross section D of the main BF vanishes. Otherwise, the rod OD, in the absence of clamping, would have a main BF of the 3rd type, which contradicts statement b) above.

II. At no value of the clamping stiffness, the deflection in the cross section C of the main BF of the rod OD vanishes. Otherwise, with some clamping stiffness, the main BF of the single-span rod OD, formed by removing the elastic support C, would have a node at C, which would imply the existence of two internal inflection points of the main BF of this rod. This would mean that its main CRF is higher than the 2nd CRF of the rod (OD). At the same time (Sec. 4.1.1), one constraint in the form of clamping introduced in the cross section D of the rod (OD) cannot raise the main CRF above the 2nd CRF of the rod (OD).

III. For any finite clamping stiffness in the cross section D, the slope of this section in the main BF of the rod OD has the same direction as the rectilinear segment CD. This follows from the fact that at zero clamping stiffness in the main BF (type 2) of the rod OD, these directions coincide, and with a continuous increase in clamping stiffness, this coincidence is preserved, because otherwise, one of statements I or II would not hold.

In consist of the rod S, the fragment OD can be considered as a rod elastically clamped on the support D, since joining the segment DL, provided that CRF of (DL) > CRF of S, at buckling is equivalent to the introduction of elastic clamping in the section D.

Statements I - III lead to the conclusion that with a small moving of the support D to the left of D^*, the main BF of the rod S can be schematically represented in the form of Fig. 4, where the arrows indicate the directions of the reactions of the supports. As can be seen from the figure, condition $R\theta > 0$ is satisfied in the section D of the rod, which, in accordance with Sec. 4.1.3, means that the main CRF of the rod S decreases when the support D moves to the left.

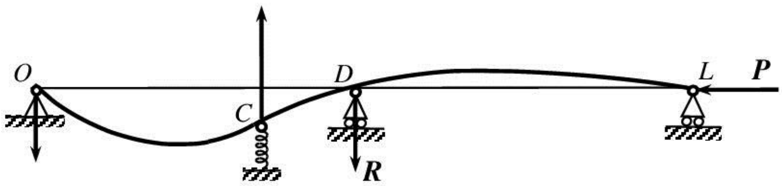

Fig. 4. BF of the rod S with a small displacement of the support D to the left from D^*

Thus, the main CRF of the rod S decreases when the support D is moved both to the right and to the left from the position D^*, which implies that when the support D is placed in the position D^*, the main CRF has a local maximum (which does not exclude the possibility of other local extrema, among which may be the desired absolute maximum of the CRF).

Let $P(D)$ be the main CRF of the rod S corresponding to the position of the support in D. If, for some position D to the left of D^*, the inequality $P(D) > P(D^*)$ was satisfied, between D and D^* there would be a position D' providing the minimum CRF, for which, according to Sec. 4.1.3, one of the equalities $\theta = 0$ or $R = 0$ would hold. The first is impossible due to the following circumstances.

If, for $\theta = 0$, the corresponding main BF was deformed on both sides of the support D', the installation of a rigid clamping on this support would preserve $P(D')$ in the spectrum (Sec. 4.1.2), but would make it double, because it would correspond to two linearly independent forms having a horizontal segment on one side of the support. This would mean the existence in the spectrum before the introduction of a rigid clamping CRF smaller than $P(D')$, i.e. $P(D')$ would not be the main one.

If the value $P(D')$ corresponded to a semi-curved BF, the part of the rod either to the left or to the right of D' would remain horizontal. If the left segment is not deformed, the right part $(D'L)$ must have a zero slope of BF in the cross section D', which is impossible, since the BFs of a hinged single-span rod $(D'L)$ are solutions of a 2nd order linear differential equation [12], for which the only solution vanishing at the boundary together with the first derivative, is the identical zero.

If the right section is not deformed, the inequality $P(D') > P(D^*)$ must be satisfied, because the corresponding BF is the BF of a rod formed from the S by imposition of constraints that prevent deformation of the section $D'L$ (Sec. 4.1.1, 4.1.2). This inequality contradicts the assumption that $P(D')$ is minimum.

Equality $R = 0$ means that D' is the node of the corresponding BF, which, taking into account the minimality of $P(D')$, should be the main one for the rod formed from S by removing the support D. But according to Sec. 4.1.5 (see Fig. 3), the main BF of such a rod does not have nodes in the segment CL.

Thus, the assumption that there is a position to the left of D^* that ensures the minimum of the main CRF of the rod S leads to a contradiction, from which it follows that as the support D moves continuously to the left from D^* to C, the CRF $P(D)$ monotonically decreases and, therefore, cannot exceed $P(D^*)$.

For any position D to the right of D^*, the inequality $P(D) > P(D^*)$ is satisfied due to the same circumstances as the inequality $P(D') > P(D^*)$ above. Thus, for all positions of the

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support D, both to the right and to the left of D^*, the relation $P(D^*) > P(D)$ is satisfied, from which the validity of the theorem follows.

Remark. The requirement to locate the support C to the left of the node A is essential. Otherwise, it can be proved that the semi-curved BF described in Sec. 4.2.1, which can exist at a sufficiently high stiffness c and for $P(OC) < P(CL)$, provides a local maximum of main CRF, which may not be absolute.

4.2.3. **Limitations**. The proved theorem describes the sought-for optimal position only under the condition that the considered CRF is the main one. This condition is satisfied, in particular, if the length of the rod S is such that the segment D^*L is more stable than OD^*, $P(D^*L) > P(D^*)$, because in this case, the loss of stability occurs due to the buckling of the segment OD^*. Elongation or shortening of the rod S due to the segment D^*L does not affect the value of its CRF, equal to $P(D^*)$, determined by the buckling of the curved segment OD^* of the semi-curved BF, however, if $\left|D^*L\right|$ exceeds a certain limit value $\left|D^*L\right|_{\lim}$, $P(D^*)$ remains in the spectrum of the CRFs of the rod S, but becomes not the main one, but one of the senior ones. For example, if the length of the rod S is such that the node A_1 of the 3rd BF of the rod (OL) is located to the right of the elastic support C, at any position of the support D, the CRF of the rod S will be less than the 3rd CRF of the rod (OL) (because the supports C and D do not fall into the nodes A_1 and A_2 of its 3rd BF, (see Sec. 4.1.2) equal to the CRF of the rod (OA_1). In this case, the CRF $P(D^*)$ of the rod S corresponding to the semi-curved BF described in Sec. 4.2.1 will be present in the spectrum of CRFs of the rod S, but will not be the main one, because according to Sec. 4.1.4 $P(D^*)>$ CRF (OC) > CRF (OA_1).

4.2.4. **Limit length**. The following considerations can be used to determine $\left|D^*L\right|_{\lim}$. Let $P(D^*)$ be a simple main CRF of the rod S. If you increase its length by moving the support L, you will find its position when its 2nd CRF, decreasing, becomes equal to $P(D^*)$, which, thus, will become double and, with a further increase in the length of the rod S, will become the second in its spectrum (due to a change in number). This position marks the limit length $\left|D^*L\right|_{\lim}$. Then, from two linearly independent BFs of the rod S corresponding to $P(D^*)$, one can form a BF having a node at D^* or at C. This BF will belong to the spectrum of the rod formed from the S by removal of the support D. As a result of the removal of the constraint, the main CRF will become smaller and $P(D^*)$ will be the second in the spectrum of the two-span rod (without support D), which corresponds to the BF with the node in D^*. Zero deflection in the section D^* of the 2nd BF of a two-span rod, compressed by force $P(D^*)$, can serve as a condition for determining the limiting length. Instead D^*, you can take the cross section C, but in a different rod, removing the support C and keeping the support D.

4.2.5. **Example**. The results of the work are illustrated by a numerical example in which the rod S (Fig. 5 *a*) has a constant bending stiffness along its length, $EJ = \text{const.}$, $c_{\text{cr}} = 2\pi^2 EJ/\ell^3$, $c = 2c_{\text{cr}}$.

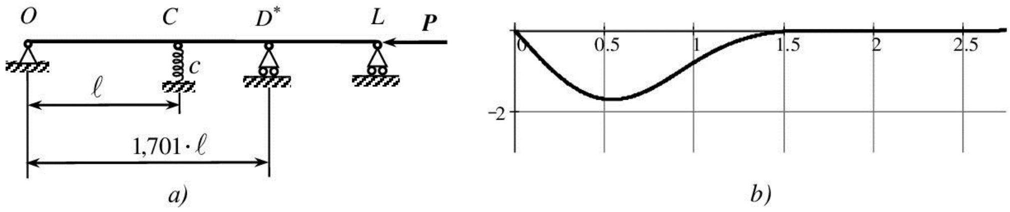

Fig. 5. Optimal prismatic rod S (*a*) and its main semi-curved BF (*b*)

As a result of the analysis of the exact equation of CRFs, got on the basis of exact expressions for the influence functions of a compressed prismatic rod [12], the optimal position of the support D was found at a distance $1{,}701 \cdot \ell$ from the left end of the rod. The corresponding value of CRF is $P_{\max} = 1{,}181 \cdot P_E$, where $P_E = \pi^2 EJ / \ell^2$, and the BF corresponding to it, normalized so that the reaction of the elastic support is equal to 1, calculated on the basis of the same expressions, is shown in Fig. 5 *b*; ordinates are given in ratio to $\ell^3 / \pi^3 EJ$, abscissas - in ratio to ℓ. This form retains its configuration and remains the main one until the distance $|DL|$ exceeds the limit value $|DL|_{\lim} = 1{,}036 \cdot \ell$ calculated from the considerations given in Sec. 4.2.4.

5 RESEARCH RESULTS DISCUSSION

The results of the work reveal some little-known aspects of the operation of longitudinally compressed rods. On their basis, answers can be obtained to a number of questions that arise during the operation and design of structures containing compressed elements. In particular, the presence of undeformed zones in the case of buckling of optimal rods gives the designer important and useful information, since in some cases it makes it possible to achieve savings due to these zones. If it is necessary to strengthen the structure, it also makes no sense to establish any constraints in these zones, since, as follows from the results of the work, these constraints will not be able to increase its critical force.

6 CONCLUSIONS

The problem of determining the position of an intermediate hinged support of a three-span rod, which ensures the maximum value of its main critical force, is considered, provided that the second intermediate support has a fixed position and finite stiffness. It is shown that the solution of the formulated problem depends on the stiffness and position of the intermediate elastic support, on the length of the rod, and also on the value and distribution of the bending stiffness, and qualitatively different solutions correspond to different combinations of these parameters. The conditions that must be satisfied by the stiffness and position of the elastic support are specified, under which the buckling of the optimal rod occurs in a semi-curved form, in which the segment of the rod on one side of the movable support remains undeformed. The results obtained allow us to better understand the behavior of compressed rods and can be used in the design and operation of engineering structures containing compressed elements.

The subject of further research should be a more complete and accurate determination of combinations of rod characteristics that provide qualitatively different solutions and the extension of conclusions to multi-span rods and more complex bar systems.

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