UDC 624.04

CALCULATION OF ANNULAR PLATES ON AN ELASTIC BASE WITH A VARIABLE BEDDING FACTOR

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Abstract. The application of the analytical method – the method of direct integration – to calculations of building structures in the form of circular plates and plates on a continuous variable elastic base is considered. It is noted that there are no proposals for a general analytical method for calculation of annular plates on a variable elastic base in the literature. And the need for such a method is obvious, since it makes it possible to estimate the accuracy of finite element analysis.

A detailed description of the algorithm of the direct integration method is not given in the paper, and all the calculation formulas for the circular plate are taken from the authors' already published article. The results of numerical implementation of this algorithm for specific examples are considered. In order to verify the results of calculations by the author's method, computer modeling of the considered circular plates in PC LIRA-SAPR and their calculations by the finite element method have been performed.

The reaction of the foundation is described by the Winkler model with a variable bedding factor. The calculation of a concrete slab that is rigidly pinched on the inner contour and articulated on the outer contour is performed. And calculation of a steel plate with rigid pinching on the outer contour and articulated on the inner contour. In the first case, the bedding factor is assumed constant, and in the second case, it changes according to the linear law. The calculations showed that the discrepancy between deflections calculated by the finite-element method and the author's method does not exceed 1%, and the results of radial and circumferential moments calculation differ more considerably, amounting to 10%. The authors explain this difference by the inaccuracy of the numerical analysis associated with a semi-automatic method of constructing a finite-element mesh, which should be made finer. The densification of the mesh in the manual mode of its partitioning significantly reduces the discrepancy between the results of calculating the deflections, radial and circumferential bending moments by the finite-element method and the author's method.

Keywords: direct integration method, annular slab, elastic foundation, Winkler model, variable bedding factor, finite element method, PC LIRA-SAPR.

РОЗРАХУНОК КІЛЬЦЕВИХ ПЛИТ НА ПРУЖНІЙ ОСНОВІ ЗІ ЗМІННИМ КОЕФІЦІЄНТОМ ПОСТЕЛІ

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Анотація. Розглядається застосування аналітичного методу – методу прямого інтегрування – до розрахунків будівельних конструкцій у формі кільцевих пластин та плит на безперервній змінній пружній основі. Зазначається, що пропозиції щодо загального аналітичного методу розрахунку кільцевих пластин на змінній пружній основі у літературі відсутні. А необхідність такого методу очевидна, оскільки дозволяє оцінити точність скінчено-елементного аналізу.

Детальний виклад алгоритму методу прямого інтегрування у роботі не наводиться, проте розрахункові формули для кільцевої пластини взяті з вже опублікованої статті авторів. Розглядаються результати чисельної реалізації цього алгоритму для конкретних прикладів. З метою верифікації результатів розрахунку авторським методом, виконано комп'ютерне моделювання розглянутих кільцевих пластин у ПК ЛІРА-САПР та їх розрахунки методом скінчених елементів.

Реакція основи описується моделлю Вінклера зі змінним коефіцієнтом постелі. Виконано розрахунок бетонної плити, яка жорстко защемлена за внутрішнім контуром і шарнірно – за зовнішнім. І розрахунок сталевої плити з жорстким затиском по зовнішньому контуру і шарнірним по внутрішньому. У першому випадку коефіцієнт постелі прийнято постійним, а у другому випадку він змінюється за лінійним законом. Виконані розрахунки показали, що розбіжність у результатах обчислення прогинів методом скінчених елементів і авторським методом не перевищує 1%, а результати обчислення радіальних і окружних моментів відрізняються значно, досягаючи 10%. Причому цю відмінність автори пояснюють неточністю чисельного аналізу, пов'язаної з напівавтоматичним методом побудови скінчено-елементної сітки, яку слід робити дрібнішою. Згущення сітки в ручному режимі її розбиття суттєво знижує розбіжність між результатами обчислення прогинів, радіальних та окружних моментів, що згинають, методом скінчених елементів і авторським методом.

Ключові слова: метод прямого інтегрування, кільцева плита, пружна основа, модель Вінклера, змінний коефіцієнт постелі, метод скінчених елементів, ПК ЛІРА-САПР.

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1 INTRODUCTION

Nowadays, the construction of circular-shaped engineering structures has become widespread. These include television towers, smoke and ventilation stacks, radio relay towers, support towers for wind power plants, and cylindrical tanks. The foundations of such structures, their floor slabs, and the bottoms of cylindrical tanks are often circular plates. And such plates are based on an elastic foundation, the reaction of which can be variable.

From a mathematical point of view, the problem of calculating such structures is very complicated, since its solution is reduced to the solution of differential equations with variable coefficients.

There are different models of elastic foundations. The simplest one is the Winkler model. For beams on a constant elastic base, this model allows us to obtain the exact solution, but there is no such solution for slabs. The problem becomes even more complicated if the elastic foundation is variable. A universal analytic solution has not yet been constructed. That is why approximate calculation methods are used, principally the finite-element method. However, it is well known that results obtained by this method are often far from reality, so the construction of an analytical solution remains relevant.

2 LITERATURE ANALYSIS AND PROBLEM FORMULATION

The main results in the construction of the theory of plate bending belong to Simeon Poisson, Louis Navier and Gustav Kirchhoff. In 1820, a report by L. Navier presented the differential equation for transverse bending of a plate in general form [1]. In 1828, in [2], for the first time was obtained the solution of the bending problem for an elastic thin axisymmetric circular plate under the action of a uniform transverse load and a concentrated transverse force in the center for two cases of boundary conditions – free resting and rigid pinching of the plate outer contour. It was in this work that Poisson's value, now called Poisson's coefficient, was introduced. Further development of the theory of plate bending is displayed in the works of Kirchhoff, who derived the variational equation for the plate, which allows to construct the differential equation and the boundary conditions of the problem.

Much research has been conducted from the theory of calculating circular and annular plates on different models of elastic foundations. Obtaining an exact solution of such problems is associated with mathematical difficulties. In this connection, numerical methods are still the only possible means of obtaining acceptable results in terms of accuracy and time consumption when solving practically important problems.

The main results obtained by scientists from various countries during the previous two centuries in the area of circular and circular plate calculation have been systematized by K. G. Chizhevsky [3]. Here are calculations of loaded circular and circular plates for strength, stiffness, stability and vibrations.

Many interesting works appeared at the end of the last century and the beginning of this century. For example, the solution of the static stability problem for a three-layer circular plate with a symmetrical cross-section structure is presented in [4]. In this paper, the system of differential equations describing the plate in question is obtained, and the finite difference method is used to determine the critical loads and the forms of stability loss. The results obtained are compared with the data of finite element analysis.

In [5], a new algorithm for calculating bending strength for reinforced concrete circular elements without taking into account the axial force is developed and a practical table for determining the areas of longitudinal reinforcement is proposed. The table has a universal character, since it can be applied to any size of section and any concrete strength class below C50.

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The plane problem of the theory of elasticity for an annular plate of a constant thickness, the middle surface of which coincides with the coordinate plane of the cylindrical coordinate system, is considered in [6]. To describe its three-dimensional stress state, three harmonic functions are used, which express the general solution of the Lyame equations in the cylindrical coordinate system. A mathematical model describing the flat stress-strain state of the plate in the polar coordinate system has been developed.

The study [7] is based on the full system of 20 equations in curvilinear non-orthogonal coordinates of linear thin shell theory, previously used in the static calculation of the long unfolding helicoid. In this paper, this system is applied to determine the stress-strain state of circular and annular plates under an external axisymmetric surface load acting both in the plate plane and from the plate plane. The results obtained for an annular plate in non-orthogonal coordinates extend the class of problems that can now be solved analytically. They can be used as the first terms of expansion series of unknown displacements in case of application of the method of small parameter to a long unfolding helicoid.

However, there are no proposals in the literature regarding a general method for the calculation of circular plates on a variable elastic base

3   RESEARCH GOAL AND OBJECTIVES

The purpose of this paper is to apply the analytical method to the calculation of building structures in the form of circular plates and slabs lying on a continuous variable elastic base.

The mentioned method is proposed in [8–10], and is called the "direct integration method".

Here the method is used to calculate an annular plate with two variants of boundary conditions: rigid pinch along the inner edge of the plate and hinged fastening of the outer edge, and vice versa – hinged fastening inside the plate with rigid fastening outside.

4   RESEARCH RESULTS

Consider a circular plate (plate) of constant cylindrical stiffness D, lying on a variable elastic base and under the action of a continuously distributed arbitrary transverse load (Fig. 1). The abbreviations adopted here are as follows: a and b are the radii of external and internal circles of the plate, r is radial coordinate $(0 \leq r \leq a)$, $q(r)$ is arbitrary transverse load, $R(r)$ is the reaction of elastic base. The internal forces acting in the plate are radial M_r and circumferential M_θ bending moments, radial transverse force Q_r.

For the bedding factor $k(r)$ and $q(r)$ loads are taken representations [12]:

$$k(r) = k_0 \left(A_0 + A_1 \left(\frac{r}{a} \right) + A_2 \left(\frac{r}{a} \right)^2 + ... + A_s \left(\frac{r}{a} \right)^s \right); \tag{1}$$

$$q(r) = q_0 \left(B_0 + B_1 \left(\frac{r}{a} \right) + B_2 \left(\frac{r}{a} \right)^2 + ... + B_p \left(\frac{r}{a} \right)^p \right), \tag{2}$$

where k_0, q_0 are the values of bedding factor and load at some characteristic point of the plate.

For the bending of circular and circular plates, the following formulas for the deflection function $w(r)$ were obtained in [12]:

$$w(r) = \frac{q_0 a^4}{D} W(r); \tag{3}$$

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$$W(r) = \lambda_1 X_1(r) + \lambda_2 X_2(r) + \lambda_3 Y_1(r) + \lambda_4 Y_2(r) + X_3(r), \qquad (4)$$

where $W(r)$ – dimensionless function and λ_n – arbitrary dimensionless constants.

A detailed description of the algorithm of the direct integration method and all the computational formulas for the circular plate are given in our paper [12]. Here we will consider the results of numerical implementation of this algorithm for several specific examples. In order to verify the results of calculations by the author's method, computer modeling of all considered examples in PC LIRA-SAPR and calculations by the finite element method have been performed.
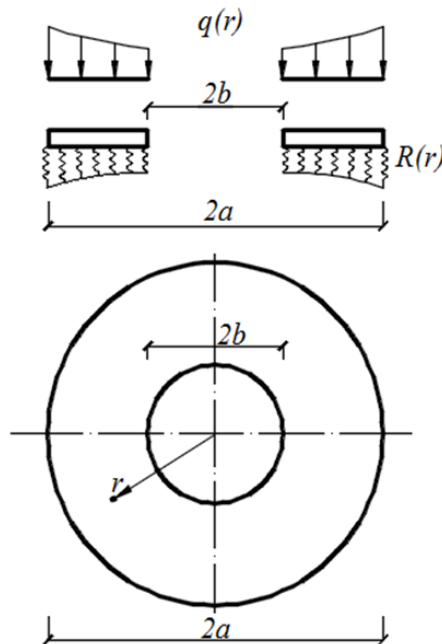
Fig. 1. Circular plate on a variable elastic base under the action of an arbitrary transverse load

Consider a concrete slab ($E = 1,5 \cdot 10^7 kPa$, $\mu = 1/6$) of thikcness $h = 0,12m$, outer radius $a = 1,8m$, inner radius $b = 0,9m$, which is under the action of a uniformly distributed constant load $q = 80kPa$. The slab is rigidly pinched along the inner contour and hinged along the outer contour.

The bedding factor (Fig. 2) is constant: $k(r) = const = 5000kH/m^3$.
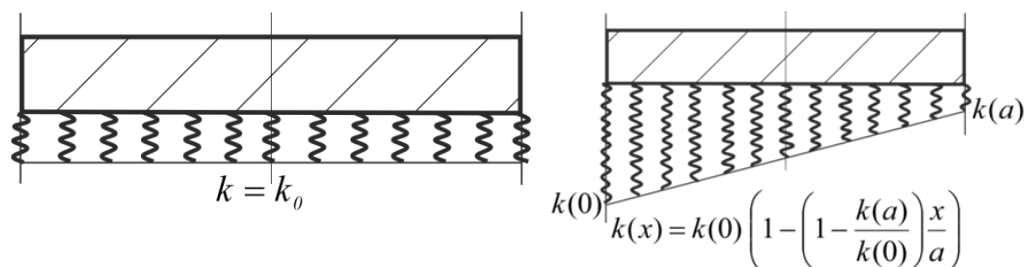
Fig. 2. Laws of change in the bed coefficient

The results of calculations by the author's method (AM) and by the finite element method (FEM) in PC LIRA-SAPR are shown in Table 1, and their graphical interpretation is shown in Fig. 3.

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Table 1

Concrete slab calculation results

r, м	w, м		Discre-pancy (%)	M_r		Discre-pancy (%)	M_θ		Discre-pancy (%)
	FEM	AM		FEM	AM		FEM	AM	
0,9	0,0000	0,0000	0,000	-10,050	-10,168	1,156	-1,6680	-1,6946	1,571
0,9429	0,0051	0,0052	0,400	-7,2124	-7,4469	3,148	-1,5852	-1,6271	2,576
0,9857	0,0186	0,0187	0,390	-4,8106	-5,0962	5,604	-1,4247	-1,4788	3,661
1,0286	0,0377	0,0379	0,380	-2,8568	-3,0756	7,113	-1,2171	-1,2754	4,574
1,0714	0,0603	0,0605	0,369	-1,2351	-1,3486	8,418	-0,9783	-1,0369	5,651
1,1143	0,0842	0,0845	0,358	0,1039	0,1144	9,154	-0,7287	-0,7789	6,450
1,1571	0,1077	0,1081	0,345	1,2051	1,3381	9,936	-0,4787	-0,5138	7,027
1,2	0,1298	0,1302	0,330	2,1082	2,3430	10,02	-0,2321	-0,2514	7,682
1,2429	0,1491	0,1495	0,313	2,8677	3,1468	8,860	0,0005	0,0006	7,911
1,2857	0,1647	0,1652	0,292	3,5693	3,7644	5,182	0,2166	0,2360	8,201
1,3286	0,1761	0,1766	0,270	4,1656	4,2088	1,026	0,4120	0,4499	8,425
1,3714	0,1832	0,1832	0,000	4,3326	4,4908	3,223	0,5833	0,6383	8,614
1,4143	0,1848	0,1847	0,218	4,4142	4,6202	4,458	0,7278	0,7980	8,789
1,4571	0,1811	0,1810	0,264	4,3408	4,6053	5,743	0,8429	0,9264	9,010
1,5	0,1723	0,1722	0,326	4,1408	4,4532	7,014	0,9322	1,0216	8,753
1,5429	0,1585	0,1584	0,345	3,8272	4,1705	8,232	0,9906	1,0819	8,440
1,5857	0,1401	0,1400	0,356	3,4159	3,7626	9,215	1,0157	1,1060	8,163
1,6286	0,1176	0,1175	0,374	2,9127	3,2345	9,995	1,0249	1,0929	6,218
1,6714	0,0915	0,0914	0,396	2,4491	2,5906	5,461	0,9948	1,0417	4,500
1,7143	0,0626	0,0625	0,409	1,8131	1,8346	1,170	0,9222	0,9519	3,125
1,7571	0,0319	0,0318	0,432	0,9174	0,9701	5,430	0,8105	0,8228	1,492
1,8	0,0000	0,0000	0,0000	0,0000	0,0000	0,000	0,6491	0,6542	0,781
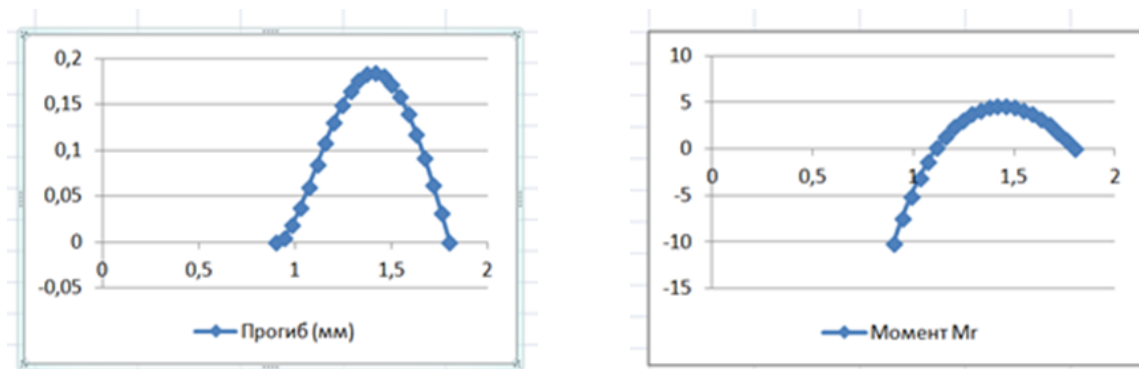
Fig.3. Deflections and radial bending moments

As follows from Table 1, the values of deflections practically coincide when calculating the ring plate under the above conditions by the author's method and with the help of the LIRA-SAPR PC [13-14], and the discrepancy of bending moments is quite significant, and even reaches 10%.

Now consider a steel plate ($E = 2,0 \cdot 10^8 \, kPa$, $\mu = 0,33$) of thickness $h = 0,03m$, outer radius $a = 0,9m$, inner radius $b = 0,45m$, which is under the action of a uniformly distributed constant load $q = 30 kPa$. The plate is rigidly pinched along the outer contour and articulated along the inner contour. The bedding coefficient varies according to the linear law: $k(b) = 4000 kH / m^3$, $k(a) = 5000 kH / m^3$ (Fig. 2).

The results of calculations by the author's method (AM) and by the finite element method (FEM) in PC LIRA-SAPR are shown in Table 2, and their graphical interpretation is shown in Fig. 4.

Table 2

Steel plate calculation results

r, м	w, м		Discre-pancy (%)	M_r		Discre-pancy (%)	M_θ		Discre-pancy (%)
	FEM	AM		FEM	AM		FEM	AM	
0,45	0,0000	0,0000	0,000	0,0000	0,0000	0,000	-0,0652	-0,0665	1,904
0,4714	0,0013	0,0013	0,120	0,0650	0,0661	1,737	-0,0384	-0,0391	1,845
0,4929	0,0026	0,0026	0,190	0,1147	0,1167	1,725	-0,0148	-0,0151	1,834
0,5143	0,0036	0,0037	0,311	0,1527	0,1549	1,427	0,0055	0,0056	1,696
0,5357	0,0046	0,0047	0,329	0,1815	0,1832	0,942	0,0232	0,0235	1,258
0,5571	0,0055	0,0056	0,358	0,2004	0,2035	1,504	0,0384	0,0387	0,848
0,5786	0,0061	0,0062	0,324	0,2103	0,2169	3,049	0,0509	0,0515	1,207
0,6	0,0065	0,0066	0,314	0,2120	0,2244	5,505	0,0610	0,0620	1,618
0,6214	0,0068	0,0069	0,303	0,2084	0,2263	7,915	0,0683	0,0701	2,574
0,6429	0,0069	0,0070	0,282	0,2024	0,2224	8,988	0,0711	0,0756	5,975
0,6643	0,0067	0,0068	0,270	0,1914	0,2127	9,993	0,0716	0,0786	8,936
0,6857	0,0065	0,0065	0,000	0,1766	0,1967	10,23	0,0708	0,0786	9,960
0,7071	0,0060	0,0060	0,001	0,1563	0,1737	10,00	0,0696	0,0754	9,861
0,7286	0,0054	0,0054	0,001	0,1298	0,1431	9,285	0,0680	0,0686	8,397
0,75	0,0045	0,0046	0,126	0,0967	0,1043	7,241	0,0543	0,0581	6,496
0,7714	0,0038	0,0038	0,005	0,0536	0,0567	5,462	0,0411	0,0433	5,120
0,7929	0,0029	0,0029	0,006	-0,0002	-0,0002	3,096	0,0237	0,0242	2,144
0,8143	0,0021	0,0021	0,004	-0,0668	-0,0669	0,196	0,0003	0,0003	1,537
0,8357	0,0013	0,0013	0,006	-0,1414	-0,1435	1,447	-0,0282	-0,0284	0,529
0,8571	0,0006	0,0006	0,009	-0,2237	-0,2301	2,768	-0,0610	-0,0620	1,580
0,8786	0,0002	0,0002	0,012	-0,3134	-0,3267	4,067	-0,0973	-0,1007	3,395
0,9	0,0000	0,0000	0,000	-0,4100	-0,4331	5,321	-0,1374	-0,1444	4,872
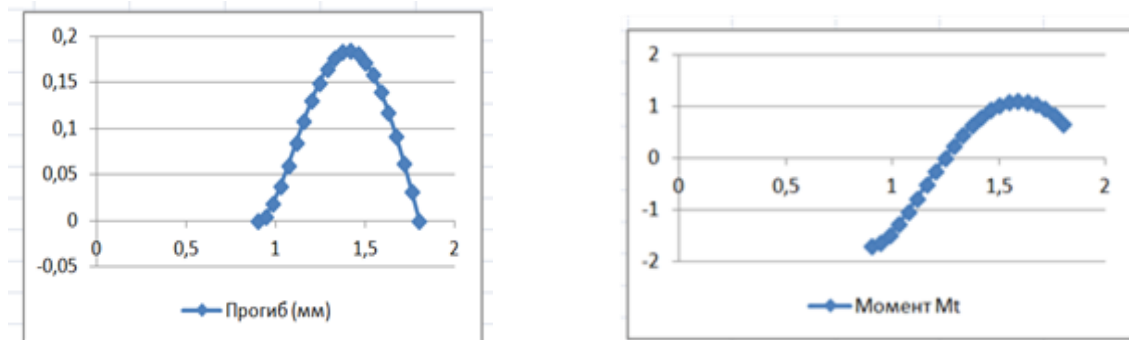
Fig.4. Deflections and circumferential bending moments

And in this case, the differences in the values of deflections calculated by the two methods are practically absent, and the differences in the values of radial and circumferential bending moments reach 10%.

5 DISCUSSION OF RESEARCH RESULTS

The results of calculations by the finite element method in PC LIRA-SAPR, which are given in Tables 1 and 2, were obtained with a semi-automatic breakdown of the finite-element grid. This means that a partitioning step of $0,1r$ was chosen along the radius, and the partitioning along the circumference of each plate was carried out automatically. With this approach, the difference in the results of calculating the deflections of FEM and the author's method was insignificant, while the difference in the results of calculating the radial and circumferential moments reaches 10%. The authors of paper [15] have shown that when the mesh is thickened in the circumferential direction, the results obtained by the two methods in determining the radial and circumferential bending moments converge considerably. Therefore, we should recommend a "manual", finer grid breakdown in finite element calculations of similar structures, regardless of the software used.

6 CONCLUSIONS

Thus, the high accuracy and efficiency of our proposed analytical method – the method of direct integration – for calculations of building structures in the form of circular plates and plates which lie on a continuous variable elastic base have been shown.

The results of calculations show that the discrepancy between the results of deflections calculation by the finite element method and the author's method does not exceed 1 %, and the results of radial and circumferential moment calculation differ more considerably, reaching 10 %. This difference is caused by inaccuracy of numerical analysis associated with the method of constructing a finite-element mesh which should be made finer. This recommendation becomes especially important if the design of a slab and its reinforcement is based on modeling and finite element analysis with the use of an engineering computer program.

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